HYDRAULIC ANALYSIS OF JUNCTIONS

BUREAU OF ENGINEERING
City of Los Angeles

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I FOREWORD AND ACKNOWLEDGMENT

The general formula for the hydraulic analysis of junctions which has been used in this monograph was derived by Donald Thompson, Chief Engineer of Design, Bureau of Engineering, City of Los Angeles. The formula is based on the well-known pressure plus momentum theory which states that the change in pressure through a junction is equal to the change in momentum. The application of the formula to actual design problems, the determination of control points, and the graphical solutions for conditions where a direct solution was not possible were prepared by Irving R. Cole, Division Engineer, Storm Drain Design Division.

Valuable assistance and advice have been given by Floyd J. Doran, Deputy City Engineer. Numerous model tests conducted over a period of several years at the Experimental Hydraulic Research Laboratory of the Bureau of Engineering have confirmed the accuracy of the Thompson formula and the pressure plus momentum theory.

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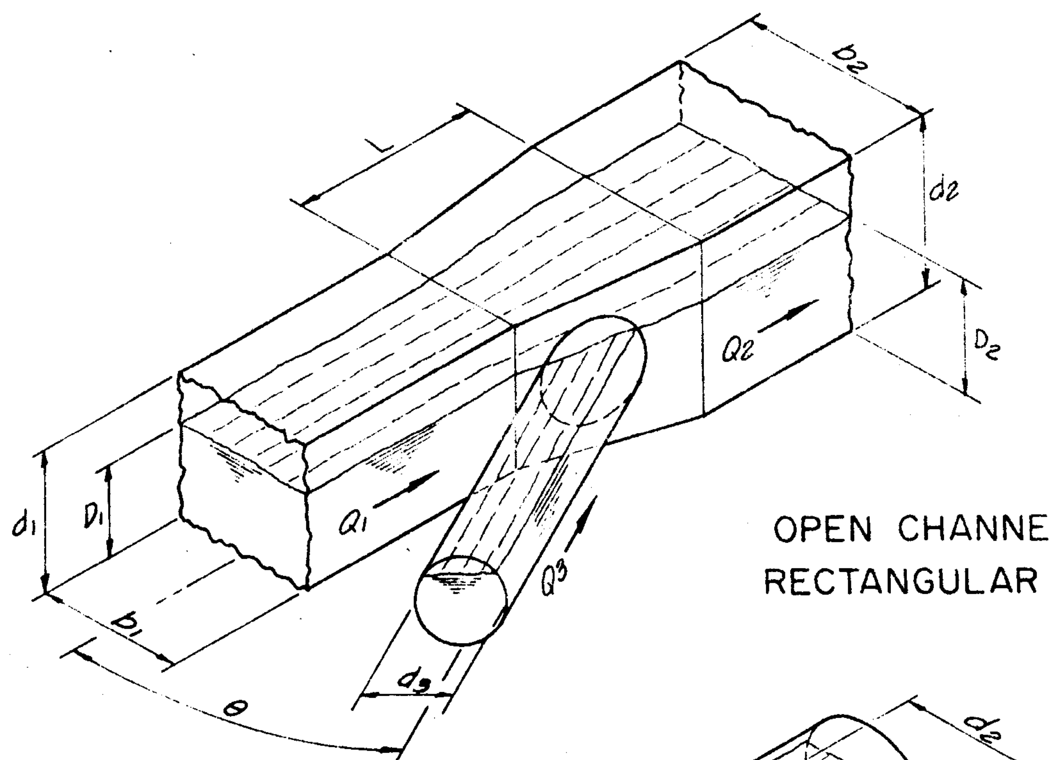
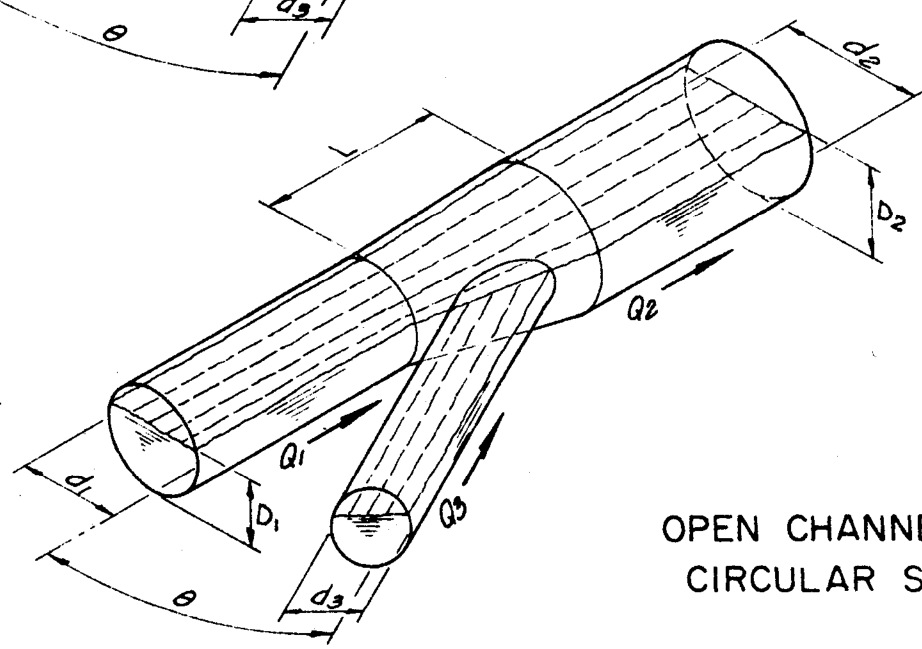


OPEN CHANNEL FLOW
RECTANGULAR SECTION



OPEN CHANNEL FLOW
CIRCULAR SECTION

II INTRODUCTION

A. Purpose and Objectives

Junctions in conduits can cause major losses in both the energy grade and the hydraulic grade across the junction. If these losses are not included in the hydraulic design, the capacity of the conduit may be seriously restricted. The pressure plus momentum theory, which equates the summation of all pressures acting at the junction with the summation of the momentums, affords a rational method of analyzing the hydraulic losses at a junction. The pressures which must be evaluated are (1) upstream end of the junction, (2) downstream end of the junction, (3) wall pressures, (4) invert pressure, and (5) soffit pressure. Formulas for the above pressures, derived from principles of hydrostatics, are extremely complicated, difficult if not impossible to remember, and, because of their complexities, may result in frequent errors. The general formula used in this monograph makes it unnecessary to evaluate individual pressures. It can be shown (see below) that, regardless of the shape of the conduit, the summation of all pressures acting at the junction, ignoring friction, is equal to the average cross-sectional area through the junction, multiplied by the change in the hydraulic gradient through the junction.

The following discussion, together with the sample problems and their solutions, illustrates the use of the general formula in determining the hydraulic changes at a junction. The discussion includes (1) the derivation of the general formula for both rectangular and circular conduits under open flow and pressure flow conditions, (2) the determinations of the control points for subcritical and supercritical flow in open channels, and (3) the solution for the hydraulic grade of the lateral under pressure flow conditions.

B. Notation

(Unit Weight of Water Omitted)

Q	Discharge, cubic feet per second (cfs.).
A	Area of flow, square feet (ft^2).
A_m	Mean Area, square feet (ft^2).
b	Width of rectangular channel, feet.
d	Diameter, of circular conduit, feet.

B. Notation (Continued)

D	Elevation of hydraulic gradient above invert, feet. Depth of flow, feet (open channel).
g	Gravitational acceleration, 32.2 feet per second per second.
Δy	Change in hydraulic gradient or water surface through the junction, feet. (Plus when increasing upstream.)
P	Hydrostatic pressure, cubic feet.
P_i	Longitudinal component of invert pressure, cubic feet.
P_s	Longitudinal component of soffit pressure, cubic feet.
P_w	Longitudinal component of wall pressure, cubic feet.
P_f	Pressure loss due to friction, cubic feet (friction loss).
V	Velocity, feet per second.
θ	Angle of convergence between the center line of the main line and the center line of the lateral, degrees.
L	Length of junction, feet.
S	Construction slope, feet per foot.
S_f	Slope of the energy gradient, feet per foot.
M	Momentum of a moving mass of water $\left(\frac{QV}{g}\right)$, cubic feet.
n	Mannings roughness coefficient.
Z	Change in invert elevation across the junction, feet.
x	Change in soffit elevation across the junction, feet.
$\bar{y}$	Distance from hydraulic gradient to center of gravity of section, feet.
h_v	Velocity head $\left(\frac{V^2}{2g}\right)$, feet.
ϕ	Angle of divergence of transition, degrees.
α	Angle of invert slope of junction, degrees.

B. Notation (Continued)

h	Energy loss, feet.
H.G.	Hydraulic gradient.
E.G.	Energy gradient.
t	Transition.
T	Top width of water surface, open channel.

Numerical subscript denotes position.
Subscript "j" denotes junction.
Subscript "c" denotes critical flow.
Subscript "n" denotes normal flow.
Subscript "tr" denotes transition.

C. General Formula

The net hydrostatic pressure at a junction equals the change in momentum through the junction plus friction.

GENERAL FORMULA WITH FRICTION INCLUDED

(UNIT WEIGHT OF WATER OMITTED)

$$P_2+M_2 = P_1+M_1+M_3\cos\theta+P_w+P_i-P_f \qquad (1)$$

$$P_1+P_w+P_i-P_2 = M_2-M_1 = M_3\cos\theta+P_f$$

$$\text{Net hydrostatic pressure} = \Sigma P = P_1+P_w+P_i-P_2$$

$$\Delta y(\text{AVERAGE AREA}) = P_1+P_w+P_i-P_2 \qquad (2)$$

$$\text{AVERAGE AREA} = {}^1/_6(A_1+4A_m+A_2)$$

$$\text{or for practical use } \tfrac{1}{2}(A_1+A_2)$$

$$\tfrac{1}{2}(A_1+A_2)\Delta y = M_2-M_1-M_3\cos\theta+P_f$$

$$= \frac{Q_2V_2-Q_1V_1-Q_3V_3\cos\theta}{g} + \frac{L(S_1+S_2)}{2}\,\frac{(A_1+A_2)}{2} \qquad (3)$$

C. General Formula (Continued)

Omitting friction, equation (3) is shown as:

$$\tfrac{1}{2}(A_1+A_2)\Delta y = \frac{Q_2V_2-Q_1V_1-Q_3V_3\cos\theta}{g} \tag{4}$$

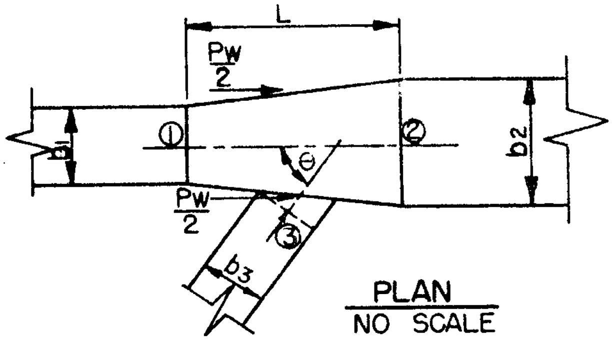
$$= \frac{Q_2{}^2}{A_2g} - \frac{Q_1{}^2}{A_1g} - \frac{Q_3{}^2\cos\theta}{A_3g} \tag{5}$$

Equations 3, 4 and 5 are valid for all types of Prismatoidal and Circular Channels and Conduits.

III OPEN CHANNEL FLOW

A. Criteria for Junction

Flow in channels and conduits with a free water surface is called open channel flow. The lateral inlet is considered to be either submerged or to have a water surface elevation approximately equal to the average water surface elevation through the junction. The hydraulic gradient or water surface elevations at Points 1 and 3 are identical.



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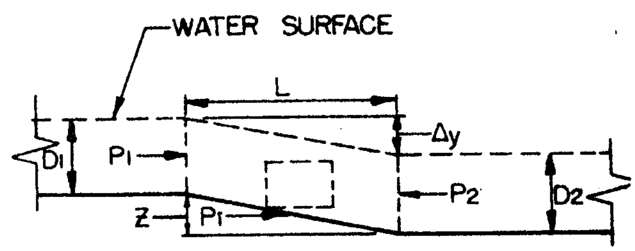

PROFILE
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B. General Formula

$$\Sigma P = \Sigma M$$

$$\tfrac{1}{2}(A_1+A_2)\Delta y = \frac{Q_2{}^2}{A_2g} - \frac{Q_1{}^2}{A_1g} - \frac{Q_3{}^2\cos\theta}{A_3g} \tag{5}$$

B. General Formula (Continued)

$$\Delta y = Z+D_1-D_2$$

$$\tfrac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2{}^2}{A_2 g} - \frac{Q_1{}^2}{A_1 g} - \frac{Q_3{}^2 \cos\theta}{A_3 g} \qquad (6)$$

C. Control Points

Flow may be either subcritical or supercritical.

1. Subcritical Flow

$$D_2 = D_{2n}$$

2. Supercritical Flow

It is necessary to determine if a hydraulic jump will occur at the junction. The depth at the downstream end of the junction (Point 2) is set at critical depth, and the momentums calculated to determine if the incoming flows can maintain supercritical flow.

a. $M_{2c} < M_{1n} + M_3 \cos\theta + \tfrac{1}{2}(A_1+A_2)\Delta y$

$$D_1 = D_{1n}$$

b. $M_{2c} > M_{1n} + M_3 \cos\theta + \tfrac{1}{2}(A_1+A_2)\Delta y$

Hydraulic jump will occur

$$D_2 = D_{2c}$$

Where the expanded structure is longer than the junction, it may be necessary to compute a drawdown water surface profile to determine the water depth and area at the point where the lateral enters the structure. Transition losses are negligible and can be ignored.

C. Control Points (Continued)

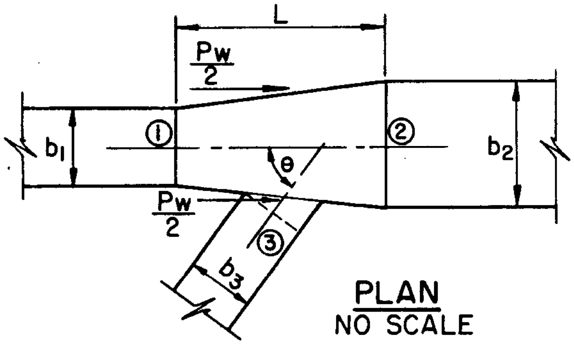
After determination of the change through the junction, backwater or drawdown calculations will have to be made to determine the water surface profile.

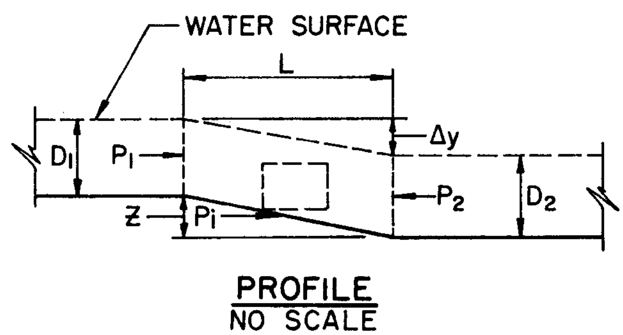
D. Derivation of Formula

1. Rectangular Section

Derivation of Equation (4) by D. Thompson.

Rectangular channel with expansion, friction ignored.





$$P_1+P_w+P_i-P_2 = \frac{Q_2V_2-Q_1V_1-Q_3V_3\cos\theta}{g}$$

$$P_1 = \tfrac{1}{2}(b_1D_1{}^2) = {}^3/_6(b_1D_1{}^2)$$

$$P_w = {}^1/_6(D_1{}^2+D_1D_2+D_2{}^2)(b_2-b_1)$$

$$P_w = {}^1/_6(b_2D_1{}^2+b_2D_1D_2+b_2D_2{}^2-b_1D_1D_2-b_1D_2{}^2-b_1D_1{}^2)$$

$$P_i = {}^1/_6(2b_1D_1+b_1D_2+b_2D_1+2b_2D_2)Z$$

$$P_2 = \tfrac{1}{2}(b_2D_2{}^2) = {}^3/_6(b_2D_2{}^2)$$

$$\Sigma P = {}^3/_6(b_1D_1{}^2)+{}^1/_6(2b_1D_1+b_1D_2+b_2D_1+2b_2D_2)Z$$

$$+ {}^1/_6(b_2D_1{}^2+b_2D_1D_2+b_2D_2{}^2-b_1D_1{}^2-b_1D_1D_2-b_1D_2{}^2)$$

$$- {}^3/_6(b_2D_2{}^2)$$

D. Derivation of Formula

$$\Delta y = Z + D_1 - D_2$$

$$\text{Average Area} = {}^1/_6(2b_1D_1 + b_1D_2 + b_2D_1 + 2b_2D_2)$$

$$\Delta y(\text{Average Area}) = {}^1/_6(2b_1D_1 + b_1D_2 + b_2D_1 + 2b_2D_2)Z$$

$$+ {}^1/_6(2b_1D_1^2 - b_1D_1D_2 + b_2D_1^2 + b_2D_1D_2 - b_1D_2^2 - 2b_2D_2^2)$$

$$\Sigma P = P_1 + P_w + P_i - P_2$$

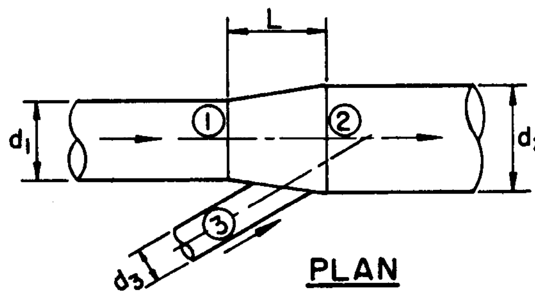
BY INSPECTION: Δy(AVERAGE AREA) = ΣP = Σ MOMENTUMS

$$\Delta y(\text{AVERAGE AREA}) = \frac{Q_2V_2 - Q_1V_1 - Q_3V_3\cos\theta}{g} \qquad (4)$$

2. Circular Section

Derivation of Equation (4) by D. Thompson.

Circular conduit with expansion, friction ignored.



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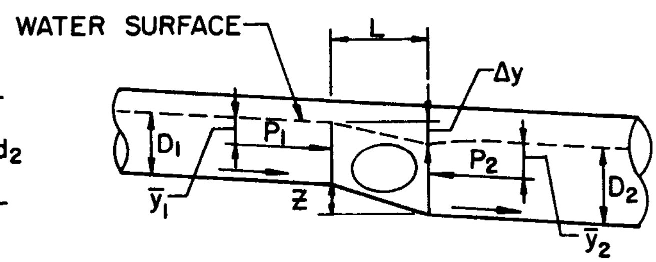

PROFILE
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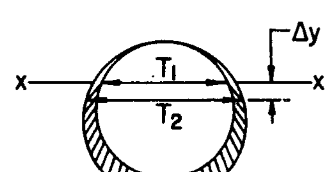
$$P_1 + P_w + P_i - P_2 = \frac{Q_2V_2 - Q_1V_1 - Q_3V_3\cos\theta}{g}$$

$$P_1 = A_1\bar{y}_1 \qquad P_w = A_w\bar{y}_w$$



SHADED AREA = A_W
VERTICAL PROJECTION
OF WALLS.

D. Derivation of Formula (Continued)

$$P_2 = A_2\bar{y}_2 \qquad P_1 = 0$$

$$A_w = A_2+\tfrac{1}{2}(T_1+T_2)\Delta y-A_1$$

$$\bar{y}_w = \bar{y}_{xx} - \tfrac{1}{2}\Delta y$$

$$\bar{y}_w = \frac{A_2(\bar{y}_2+\Delta y)+\tfrac{1}{2}T_1\Delta y^2+\tfrac{1}{2}(T_2-T_1)(\tfrac{1}{2}\Delta y)(^2/_3\Delta y)2}{A_2+\tfrac{1}{2}(T_1+T_2)\Delta y-A_1}$$

$$- \frac{A_1\bar{y}_1}{A_2+\tfrac{1}{2}(T_1+T_2)\Delta y-A_1} - \tfrac{1}{2}\Delta y$$

$$\bar{y}_w = \frac{A_2(\bar{y}_2+\Delta y)+{}^1/_3\Delta y^2(\tfrac{1}{2}T_1+T_2)-A_1\bar{y}_1}{A_2+\tfrac{1}{2}(T_1+T_2)\Delta y-A_1} - \tfrac{1}{2}\Delta y$$

$$P_w = A_w\bar{y}_w = A_2(\bar{y}_2+\Delta y)-A_1\bar{y}_1+{}^1/_3\Delta y^2(\tfrac{1}{2}T_1+T_2)$$

$$- \tfrac{1}{2}\Delta y\left[A_2+\tfrac{1}{2}(T_1+T_2)\Delta y-A_1\right]$$

$$P_w = A_2\bar{y}_2-A_1\bar{y}_1+\tfrac{1}{2}(A_1+A_2)\Delta y({}^1/_{12}\Delta y^2)(T_2-T_1)$$

${}^1/_{12}\Delta y^2(T_2-T_1)$ will be small and can be omitted.

$$\Sigma P = P_1+P_w-P_2 = A_1\bar{y}_1+A_2\bar{y}_2-A_1\bar{y}_1+\tfrac{1}{2}(A_1+A_2)\Delta y-A_2\bar{y}_2$$

$$\Sigma P = \tfrac{1}{2}(A_1+A_2)\Delta y = \Delta y(\text{Average Area}) = \Sigma \text{ Momentums}$$

$$\Delta y(\text{AVERAGE AREA}) = \frac{Q_2V_2-Q_1V_1-Q_3V_3\cos\theta}{g} \tag{4}$$

E. Outline of Examples

1. Rectangular Section - Subcritical Flow

Case A: Determine Z so that $D_1 = D_{1n}$

E. Outline of Examples (Continued)

Case B: Determine D_1 when $Z = 0$

2. Rectangular Section - Supercritical Flow

Case C: Determine Z when $D_1 = D_{1n}$ so that a hydraulic jump cannot form.

$M_2 \leqq M_{1n}+M_3\cos\theta+\frac{1}{2}(A_1+A_2)\Delta y$

$M_2 = M_{2c}$

$M_2 = M_{2n}$

Case D: Determine D_1 when $Z = 0$ and $D_2 = D_{2c}$

$M_{2c}>M_{1n}+M_3\cos\theta+\frac{1}{2}(A_1+A_2)\Delta y$

Case E: Determine D_2 when $Z = 0$ and $D_1 = D_{1n}$

$M_{2c}<M_{1n}+M_3\cos\theta+\frac{1}{2}(A_1+A_2)\Delta y$

3. Circular Section - Subcritical Flow

Case F: Determine D_1 when $Z = d_2-d_1 = 0.5'$

Case G: Determine Z so that $D_1 = {}^3/_4 d_1$ and $D_2 = {}^3/_4 d_2$

4. Circular Section - Supercritical Flow

Case H: Determine D_1 when $Z = d_2-d_1 = 0.5'$ and $D_2 = D_{2c}$

$M_{2c}>M_{1n}+M_3\cos\theta+\frac{1}{2}(A_1+A_2)\Delta y$

Case I: Determine Z so that a hydraulic jump cannot form.

E. Outline of Examples (Continued)

$$M_2 \lesseqgtr M_{1n}+M_3 \cos\theta+\tfrac{1}{2}(A_1+A_2)\Delta y$$

$$M_2 = M_{2c}$$

$$M_2 = M_{2n}$$

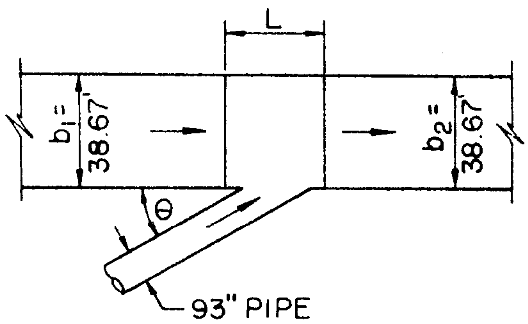
Case J: Determine D_2 when $D_1 = D_{1n}$ and $Z = 0.75'$

$$M_{2c}<M_{1n}+M_3 \cos\theta+\tfrac{1}{2}(A_1+A_2)\Delta y$$

F. Examples

1. Rectangular Section - Subcritical Flow

Case A: When flow is subcritical both upstream and downstream, $D_2 = D_{2n}$; SET $D_1 = D_{1n}$ and solve for value of Z required to maintain normal flow.



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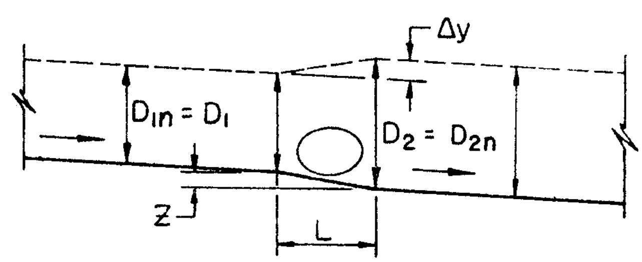

PROFILE
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GIVEN: Q_1 = 11,015 cfs	Q_2 = 11,450 cfs	Q_3 = 435 cfs
b_1 = 38.67 ft.	b_2 = 38.67 ft.	d_3 = 93 in.
S_1 = 0.0022	S_2 = 0.0020	A_3 = 47 sq.ft.
D_{1n} = 14.1 ft.	D_{2n} = 15.0 ft.	θ = 30°
A_{1n} = 545 sq.ft.	A_{2n} = 582 sq.ft.	n = 0.014

F. Examples (Continued)

FORMULA AND SOLUTION:

$$\Sigma P = \Sigma M$$

$$\tfrac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2{}^2}{A_2 g} - \frac{Q_1{}^2}{A_1 g} - \frac{Q_3{}^2 \cos\theta}{A_3 g}$$

$$\tfrac{1}{2}(545+582)(Z+14.1-15.0) = \frac{(11,450)^2}{582(32.2)} - \frac{(11,015)^2}{545(32.2)} - \frac{(435)^2(.866)}{47(32.2)}$$

$$563.5(Z-0.9) = 6990-6940-108$$

$$Z = 0.9 - \frac{58}{563.5}$$

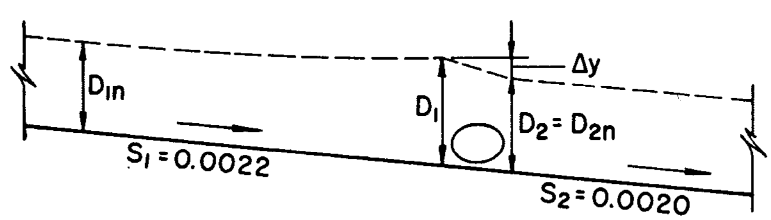
$$\underline{\underline{Z = 0.80 \text{ ft.}}}$$

$$\Delta y = Z+D_1-D_2$$

$$= 0.80+14.1-15.0$$

$$= -0.10 \text{ ft.}$$

Case B: When Z is limited or is set a certain value; $D_2 = D_{2n}$, DETERMINE D_1 when Z = 0.



PROFILE
NO SCALE

F. Examples (Continued)

GIVEN: Q_1 = 11,015 cfs Q_2 = 11,450 cfs Q_3 = 435 cfs
b_1 = 38.67 ft. b_2 = 38.67 ft. d_3 = 93 in.
S_1 = 0.0022 S_2 = 0.0020 A_3 = 47 sq.ft.
D_{1n} = 14.1 ft. D_{2n} = 15.0 ft. θ = 30°
A_{2n} = 582 sq.ft. n = 0.014

FORMULA:

$$\Sigma P = \Sigma M$$

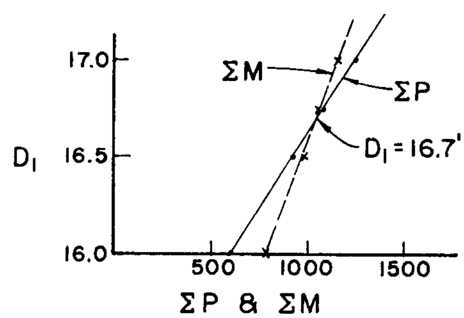
$$\tfrac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2{}^2}{A_2 g} - \frac{Q_1{}^2}{A_1 g} - \frac{Q_3{}^2 \cos\theta}{A_3 g}$$

$$\tfrac{1}{2}(A_1+582)(Z+D_1-15.0) = 6990 - \frac{3,775,000}{A_1} - 108$$

$$\tfrac{1}{2}(A_1+582)(D_1-15.0) = 6882 - \frac{3,775,000}{A_1}$$

SOLUTION:

D_1	A_1	ΣP	ΣM
16.0	620	601	790
16.5	639	918	970
16.75	648	1075	1050
17.0	657	1240	1140



PLOT D_1 VERSUS ΣP and ΣM:

D_1 = 16.7 ft.
$\Delta y = Z+D_1-D_2 = 0+16.7-15.0 =$ 1.7 ft.

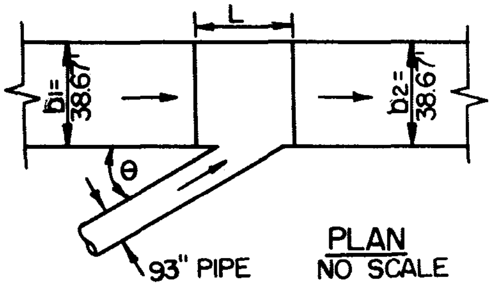
To complete the computation, determine water surface profile from D_1 = 16.7 ft. to D_{1n} = 14.1 ft.

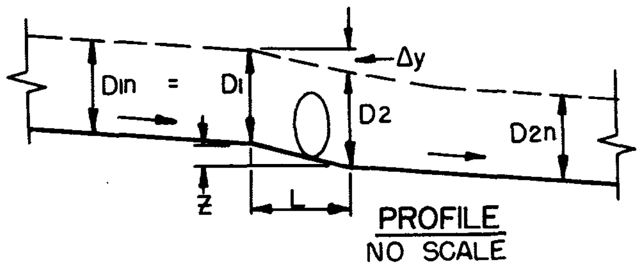
2. Rectangular Section - Supercritical Flow

Case C: Supercritical flow upstream and downstream; solve for value of Z so that a hydraulic jump

F. Examples (Continued)

cannot occur at the junction. SET $D_1 = D_{1n}$, MAXIMUM $D_2 = D_{2c}$.





GIVEN:

$Q_1 = 11{,}015$ cfs	$Q_2 = 11{,}450$ cfs	$Q_3 = 435$ cfs
$b_1 = 38.67$ ft.	$b_2 = 38.67$ ft.	$d_3 = 93$ in.
$S_1 = 0.00357$	$S_2 = 0.00357$	$A_3 = 47$ sq.ft.
$D_{1n} = 11.90$ ft.	$D_{2c} = 13.98$ ft.	$\theta = 30^\circ$
$A_{1n} = 460$ sq.ft.	$A_{2c} = 540$ sq.ft.	$n = 0.014$
	$D_{2n} = 12.22$ ft.	

FORMULA AND SOLUTION:

$$M_{2c} \leqq M_{1n} + M_3 \cos\theta + \tfrac{1}{2}(A_{1n} + A_{2c})(Z + D_{1n} - D_{2c})$$

$$\frac{(11{,}450)^2}{540(32.2)} = \frac{(11{,}015)^2}{460(32.2)} + \frac{(435)^2(.866)}{47(32.2)}$$

$$+\tfrac{1}{2}(460+540)(Z+11.90-13.98)$$

$$7540 \leqq 8210 + 108 + (500)(Z-2.08)$$

$$-778 \leqq (500)(Z-2.08)$$

$$Z \geqq 0.52 \text{ ft.}$$

ALTERNATE SOLUTION: CONSIDER $D_2 = D_{2n} = 12.22$ ft.

$$A_{2n} = 473 \text{ sq.ft.}$$

$$M_{2n} = M_{1n} + M_3 \cos\theta + \tfrac{1}{2}(A_{1n} + A_{2n})(Z + D_{1n} - D_{2n})$$

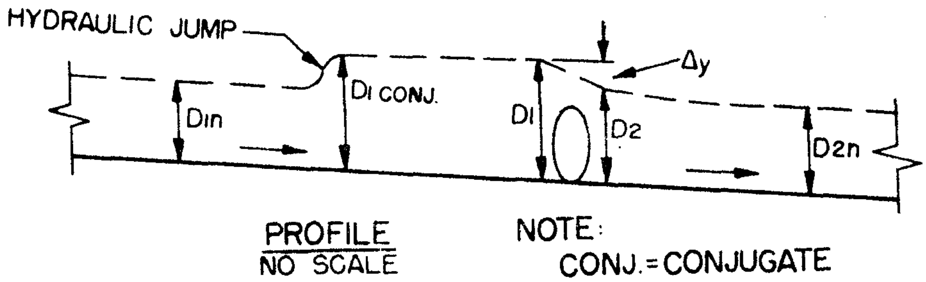
F. Examples (Continued)

$$8620 = 8210+108+466Z-149$$

$$Z = 0.97 \text{ ft.}$$

$$\Delta y = Z+D_{1n}-D_{2n} = 0.97+11.90-12.22 = \underline{\underline{0.65 \text{ ft.}}}$$

Case D: Conditions are identical to those for Case C, except that $Z = 0$; DETERMINE D_1 and D_2.



DETERMINE POINT OF CONTROL:

$$M_{2c} = 7540$$

$$M_{1n}+M_3\cos\theta+\tfrac{1}{2}(A_{1n}+A_{2c})(Z+D_1-D_2) =$$

$$8210+108+500(-2.08) = 7278$$

M_{2c} is larger $(7540>7278)$

A HYDRAULIC JUMP WILL FORM UPSTREAM OF THE JUNCTION.

$D_2 = D_{2c} = 13.98$ ft.; DETERMINE D_1.

FORMULA:

$$\Sigma P = \Sigma M$$

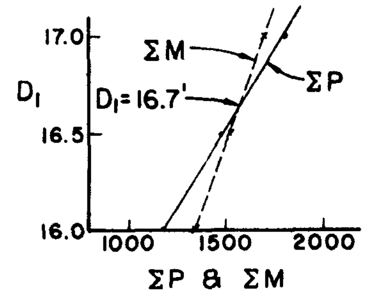
$$\tfrac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2^2}{A_2 g} - \frac{Q_1^2}{A_1 g} - \frac{Q_3^2\cos\theta}{A_3 g}$$

F. Examples (Continued)

$$\tfrac{1}{2}(A_1+540)(0+D_1-13.98) = 7540 - \frac{3,775,000}{A_1} - 108$$

SOLUTION:

D_1	A_1	ΣP	ΣM
16.0	619	1171	1330
16.5	638	1482	1520
17.0	659	1808	1690



PLOT D_1 VERSUS ΣP and ΣM:

$D_1 = 16.7$ ft.

$\Delta y = Z+D_1-D_2 = 0+16.7-13.98 = 2.72$ ft.

$D_{1n} = 11.90$ ft., $D_{2n} = 12.22$ ft.

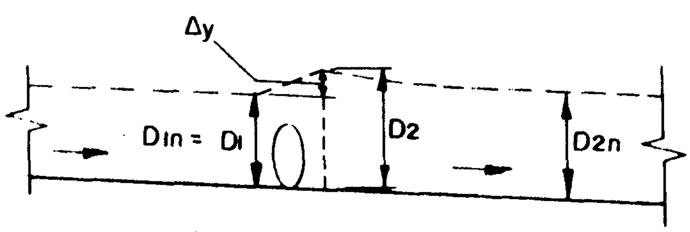
$$D_{1Conj.} = -\tfrac{1}{2}D_{1n}+\sqrt{\tfrac{1}{4}(D_{1n})^2+\frac{2Q_1{}^2(D_{1n})}{A_{1n}{}^2 g}} = 15.55 \text{ ft.}$$

DETERMINE WATER SURFACE PROFILE FOR:

(a) D_1(16.7 ft.) TO $D_{1Conj.}$(15.55 ft.)

(b) D_{2c}(13.98 ft.) TO D_{2n} (12.22 ft.)

Case E: Supercritical flow upstream and downstream, Z=0.



PROFILE
NO SCALE

F. Examples (Continued)

GIVEN:

Q_1 = 11,015 cfs	Q_2 = 11,450 cfs	Q_3 = 435 cfs
b_1 = 38.67 ft.	b_2 = 38.67 ft.	d_3 = 93 in.
S_1 = 0.00582	S_2 = 0.00582	A_3 = 47 sq.ft.
D_{1n} = 10.0 ft.	D_{2c} = 13.98 ft.	$\Theta = 30^\circ$
A_1 = 387 sq.ft.	A_2 = 540 sq.ft.	n = 0.014
	D_{2n} = 10.25 ft.	

DETERMINE POINT OF CONTROL:

$M_{2c} = 7540$

$$M_{1n}+M_3\cos\Theta+\tfrac{1}{2}(A_{1n}+A_{2c})(Z+D_{1n}-D_{2c}) = 9770+108-1840$$

$$= \underline{\underline{8038}}$$

M_{2c} is the lower value;

$D_1 = D_{1n} = 10.0$ ft.; DETERMINE D_2.

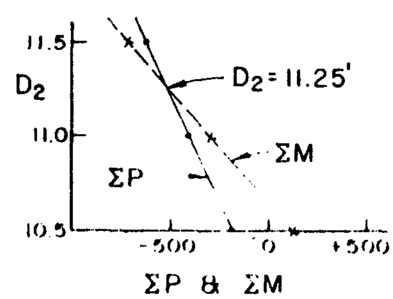
FORMULA:

$$\tfrac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2^2}{A_2 g} - \frac{Q_1^2}{A_1 g} - \frac{Q_3^2\cos\Theta}{A_3 g}$$

$$(193+\tfrac{1}{2}A_2)(10.0-D_2) = \frac{4{,}070{,}000}{A_2} - 9880$$

SOLUTION:

D_2	A_2	ΣP	ΣM
10.5	406	-198	120
11.0	425	-406	-290
11.5	445	-624	-730



PLOT D_2 VERSUS ΣP and ΣM:

$D_2 = 11.25$ ft.

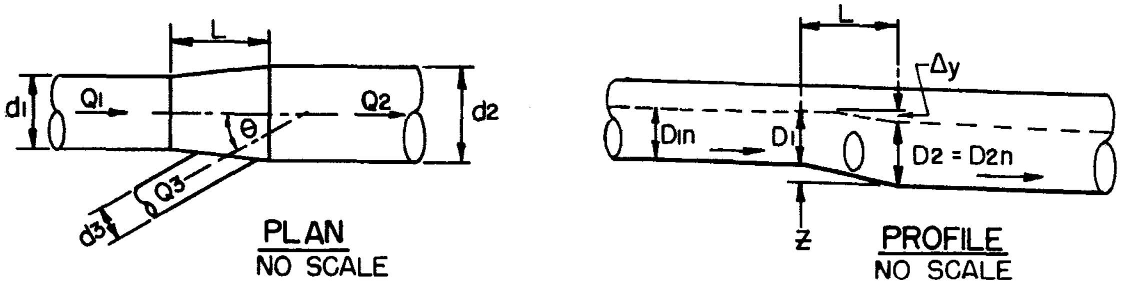
$\Delta y = Z+D_1-D_2 = 0+10.0-11.25 = 1.25$ ft.

F. Examples (Continued)

DETERMINE WATER SURFACE PROFILE FROM D_2(11.25 ft.) to D_{2n}(10.25 ft.).

3. Circular Section - Subcritical Flow

Case F: Flow is subcritical upstream and downstream, $Z = 0.5$ ft., $D_2 = D_{2n}$; DETERMINE D_1.



GIVEN:	$Q_1 = 200$ cfs	$Q_2 = 250$ cfs	$Q_3 = 50$ cfs
	$d_1 = 5.5$ ft.	$d_2 = 6.0$ ft.	$d_3 = 2.5$ ft.
	$S_1 = 0.0044$	$S_2 = 0.0036$	$A_3 = 4.91$ sq.ft.
	$D_{1n} = 3.99$ ft.	$D_{2n} = 4.83$ ft.	$\theta = 30^\circ$
		$A_{2n} = 24.4$ sq.ft.	$n = 0.013$

FORMULA:

$$\Sigma P = \Sigma M$$

$$\tfrac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2^2}{A_2 g} - \frac{Q_1^2}{A_1 g} - \frac{Q_3^2 \cos\theta}{A_3 g}$$

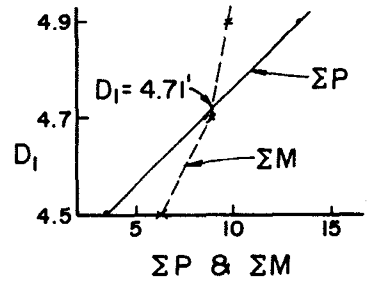
$$\tfrac{1}{2}(A_1+24.4)(0.5+D_1-4.83) = \frac{(250)^2}{24.4g} - \frac{(200)^2}{A_1 g} - \frac{(50)^2(.866)}{4.91g}$$

$$(\tfrac{1}{2}A_1+12.2)(D_1-4.33) = 79.6 - \frac{1242}{A_1} - 13.7$$

F. Examples (Continued)

SOLUTION:

D_1	A_1	ΣP	ΣM
4.5	20.85	3.85	6.2
4.7	21.7	8.53	8.7
4.9	22.2	13.28	9.8



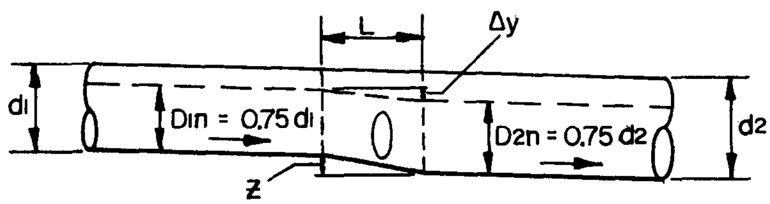
PLOT D_1 VERSUS ΣP and ΣM:

$D_1 = 4.71$ ft.

$\Delta y = Z + D_1 - D_2 = 0.5 + 4.71 - 4.83 = 0.38$ ft.

Case G: Determine required invert slopes and Z value to maintain 3/4 depth of flow:

$$\frac{D_1}{d_1} = \frac{D_2}{d_2} = 0.75$$



PROFILE
NO SCALE

GIVEN: $Q_1 = 200$ cfs, $Q_2 = 250$ cfs, $Q_3 = 50$ cfs
$d_1 = 5.5$ ft., $d_2 = 6.0$ ft., $d_3 = 2.5$
$D_1 = 4.13$ ft., $D_2 = 4.50$ ft., $A_3 = 4.91$ sq.ft.
$A_1 = 19.1$ sq.ft., $A_2 = 22.8$ sq.ft., $\theta = 30^\circ$
$n = 0.013$

USING MANNINGS EQUATION, $S_1 = 0.00428$, $S_2 = 0.00419$.

F. Examples (Continued)

FORMULA AND SOLUTION:

$$\Sigma P = \Sigma M$$

$$\tfrac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2{}^2}{A_2 g} - \frac{Q_1{}^2}{A_1 g} - \frac{Q_3 \text{Cos}\,\theta}{A_3 g}$$

$$\tfrac{1}{2}(19.1+22.8)(Z+4.13-4.50) = \frac{(250)^2}{22.8g} - \frac{(200)^2}{19.1g} - \frac{(50)^2(.866)}{4.91g}$$

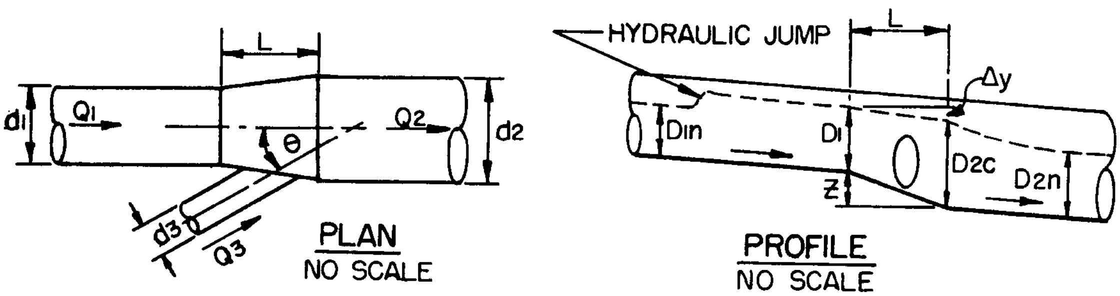
$$(21.0)(Z-0.37) = 85.2-65.0-13.7$$

$$\underline{\underline{Z = 0.68 \text{ ft.}}}$$

$$\Delta y = Z+D_1-D_2 = 0.68+4.13-4.50 = \underline{\underline{0.31 \text{ ft.}}}$$

4. Circular Section - Supercritical Flow

Case H: Supercritical flow upstream and downstream
$Z = 0.5$ ft.



GIVEN: Q_1 = 200 cfs	Q_2 = 250 cfs	Q_3 = 50 cfs
d_1 = 5.5 ft.	d_2 = 6.0 sq.ft.	d_3 = 2.5 ft.
S_1 = 0.0055	S_2 = 0.0050	A_3 = 4.91 sq.ft.
D_{1n} = 3.71 ft.	D_{2c} = 4.33 ft.	θ = 30^o
A_{1n} = 17.1 sq.ft.	A_{2c} = 21.8 sq.ft.	n = 0.013
	D_{2n} = 4.17 ft.	

F. Examples (Continued)

$$M_{2c} = \frac{Q_2^2}{A_{2c}g} = \frac{(250)^2}{21.8g} = \underline{89.0}$$

$$M_{1n}+M_3\cos\theta+\tfrac{1}{2}(A_{1n}+A_{2c})(Z+D_{1n}-D_{2c}) = \frac{(200)^2}{17.1g}$$

$$+\frac{(50)^2(.866)}{4.91g}+\tfrac{1}{2}(17.1+21.8)(0.5+3.71-4.33)$$

$$= 84.1$$

M_{2c} is larger (89>84.1)

A HYDRAULIC JUMP WILL FORM UPSTREAM OF THE JUNCTION.

$D_2 = D_{2c}$, DETERMINE D_1.

FORMULA:

$$\Sigma P = \Sigma M$$

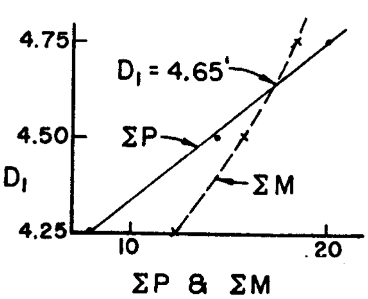
$$\tfrac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2^2}{A_2 g} - \frac{Q_1^2}{A_1 g} - \frac{Q_3^2\cos\theta}{A_3 g}$$

$$(\tfrac{1}{2}A_1+10.9)(D_1-3.83) = 75.3 - \frac{1242}{A_1}$$

SOLUTION:

D_1	A_1	ΣP	ΣM
4.25	19.7	8.7	12.2
4.50	20.8	14.3	15.7
4.75	21.8	20.0	18.3

PLOT D_1 VERSUS ΣP and ΣM:

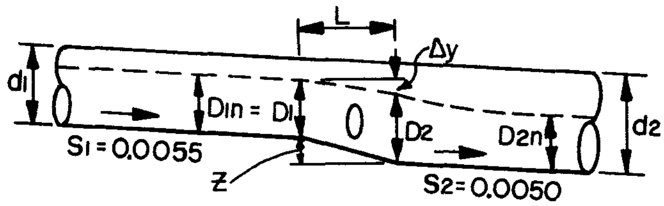


$\underline{\underline{D_1 = 4.65 \text{ ft.}}}$

$$\Delta y = Z+D_1-D_{2c} = 0.5+4.65-4.33 = \underline{\underline{0.82 \text{ ft.}}}$$

F. Examples (Continued)

Case I: Solve for the value of Z so that a hydraulic jump cannot occur. $D_1 = D_{1n}$, Maximum $D_2 = D_{2c}$.



PROFILE
NO SCALE

GIVEN: Conditions are identical to those for Case H except that Z is unknown.

FORMULA AND SOLUTION:

$$M_{2c} \leqq M_{1n}+M_3\cos\theta+\tfrac{1}{2}(A_{1n}+A_{2c})(Z+D_{1n}-D_{2c})$$

$$\frac{Q_2{}^2}{A_{2c}g} \leqq \frac{Q_1{}^2}{A_{1n}g}+\frac{Q_3{}^2\cos\theta}{A_3 g}+\tfrac{1}{2}(A_{1n}+A_{2c})(Z+D_{1n}-D_{2c})$$

$$89.0 \leqq 72.7+13.7+\tfrac{1}{2}(17.1+21.8)(Z+3.71-4.33)$$

$$\underline{\underline{Z \geqq 0.75 \text{ ft.}}}$$

ALTERNATE SOLUTION: CONSIDER $D_2 = D_{2n} = 4.17$ ft.,

$A_{2n} = 21.0$ sq.ft.

$$M_{2n} = M_{1n}+M_3\cos\theta+\tfrac{1}{2}(A_{1n}+A_{2n})(Z+D_{1n}-D_{2n})$$

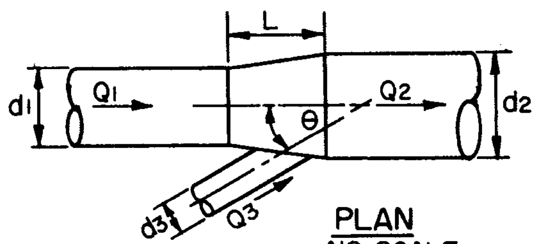
$$92.4 = 72.7+13.7+19.0Z-8.8$$

$$Z = 0.78 \text{ ft.}$$

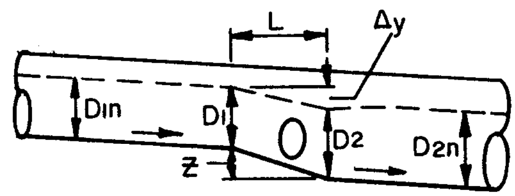
$$\Delta y = Z+D_{1n}-D_{2n} = 0.78+3.71-4.17 = \underline{\underline{0.32 \text{ ft.}}}$$

F. Examples (Continued)

Case J: Supercritical flow upstream and downstream; $Z = 0.75$ ft.



PLAN
NO SCALE



PROFILE
NO SCALE

GIVEN:

$Q_1 = 200$ cfs	$Q_2 = 250$ cfs	$Q_3 = 50$ cfs
$d_1 = 5.5$ ft.	$d_2 = 6.0$ ft.	$d_3 = 2.5$ ft.
$S_1 = 0.0065$	$S_2 = 0.0050$	$A_3 = 4.91$ sq.ft.
$D_{1n} = 3.50$ ft.	$D_{2c} = 4.33$ ft.	$\theta = 30^\circ$
$A_{1n} = 15.95$ sq.ft.	$A_{2c} = 21.8$ sq.ft.	$n = 0.013$
	$D_{2n} = 4.17$ ft.	

DETERMINE POINT OF CONTROL:

$M_{2c} = 89.0$

$M_{1n} + M_3 \cos\theta + \frac{1}{2}(A_{1n} + A_{2c})(Z + D_{1n} - D_{2c})$

$$= \frac{(200)^2}{15.95g} + \frac{(50)^2(.866)}{4.91g}$$

$$+ \tfrac{1}{2}(15.95 + 21.8)(0.75 + 3.50 - 4.33)$$

$= 90.2$

M_{2c} is smaller $(89 < 90.2)$

$D_1 = D_{1n}$, DETERMINE D_2.

F. Examples (Continued)

FORMULA:

$$\Sigma P = \Sigma M$$

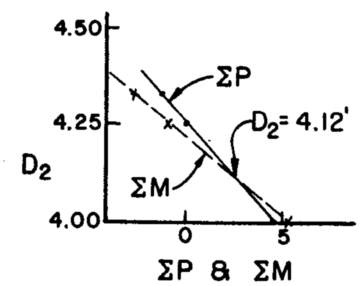
$$\tfrac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2^{\,2}}{A_2 g} - \frac{Q_1^{\,2}}{A_1 g} - \frac{Q_3^{\,2}\cos\theta}{A_3 g}$$

$$(8.0+\tfrac{1}{2}A_2)(4.25-D_2) = \frac{1940}{A_2} - 91.7$$

SOLUTION:

D_2	A_2	ΣP	ΣM
4.00	20.0	4.50	5.3
4.25	21.4	0.00	-1.0
4.33	21.8	-1.30	-2.7

PLOT D_2 VERSUS ΣP and ΣM:



D_2 = 4.12 ft.

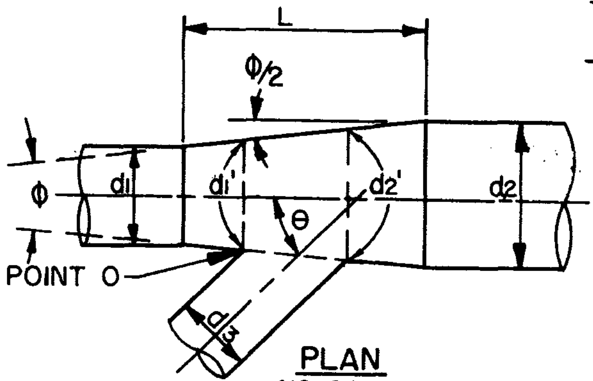
$\Delta y = Z+D_1-D_2 = 0.75+3.50-4.12 =$ 0.13 ft.

IV PRESSURE FLOW

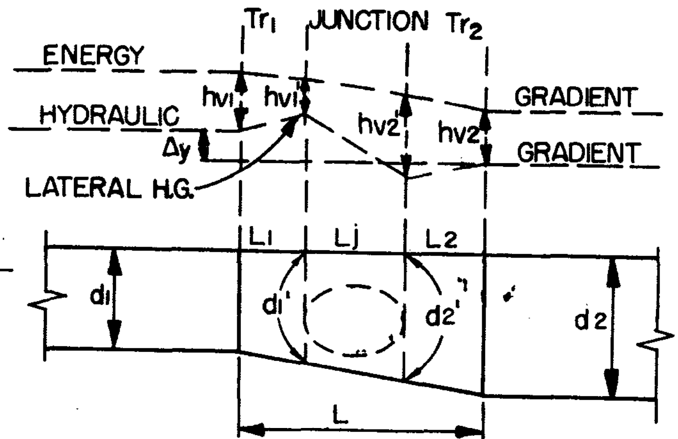
A. Criteria for Junction

A junction structure as generally constructed includes a junction plus transition structures on either side. The loss across the structure will include transition losses as well as the junction loss. The hydraulic grade for the lateral is the same as for the main line where the lateral joins the upstream end of the main line (Point 0). The transition losses are very minor and the junction structure losses may be evaluated by using the physical properties at the ends of the junction structure.

A. Criteria for Junction (Continued)



PLAN
NO SCALE



PROFILE
NO SCALE

Tr_1 and Tr_2 are transition sections either side of junction.

B. General Conditions

Junction Loss

$$\Delta y(\text{AVERAGE AREA}) = Q_2V_2'-Q_1V_1'-Q_3V_3\text{Cos}\theta$$

$$+\tfrac{1}{2}(A_1'+A_2')L_j\left[\tfrac{1}{2}(S_{f1}'+S_{f2}')L_j\right]$$

$$\Delta y_j = \frac{Q_2V_2'-Q_1V_1'-Q_3V_3'\text{Cos}\theta}{\tfrac{1}{2}g(A_1'+A_2')} + \tfrac{1}{2}L_j(S_{f1}'+S_{f2}')$$

$$h_j = \Delta y+h_{v1}'-h_{v2}'$$

Transition Loss (Enlargers)

Based on tests by Gibson (Standards of the Hydraulic Institute).

$$h_{tr} = \frac{K(V_1-V_2)^2}{2g}$$

$$K = 3.50\ (\text{TAN}\ \phi/2)^{1.22}$$

B. General Conditions (Continued)

For $\phi \cong 11^{\circ}30'$

$\tan \phi/2 = 0.100$

$K = 0.211$

$$\frac{K}{64.4} = \frac{0.211}{64.4} \cong 0.0032 \text{ for } \phi \cong 11^{\circ}30'$$

$$h_{tr1} = 0.0032\ (V_1-V_1')^2+\tfrac{1}{2}(S_{f1}+S_{f1}')L_1$$

$$h_{tr2} = 0.0032\ (V_2'-V_2)^2+\tfrac{1}{2}(S_{f2}+S_{f2}')L_2$$

Junction Structure Loss

$$h_{STRUCT} = h_j+h_{tr1}+h_{tr2}$$

$$= \Delta y+h_{v1}'-h_{v2}'+h_{tr1}+h_{tr2}$$

Since h_{tr} values are small, the transitions can be ignored.

Use all values at d_1 and d_2.

$$\Delta y = \frac{Q_2V_2-Q_1V_1-Q_3V_3\cos\theta}{\tfrac{1}{2}(A_1+A_2)g} + \tfrac{1}{2}(S_{f1}+S_{f2})L$$

$$h_{STRUCT} = \Delta y+h_{v1}-h_{v2}$$

To determine lateral hydraulic grade

$$\text{H.G. Lateral} = \text{H.G.}(1)+h_{v1}-h_{v1}'-(S_{f1}')L_1$$

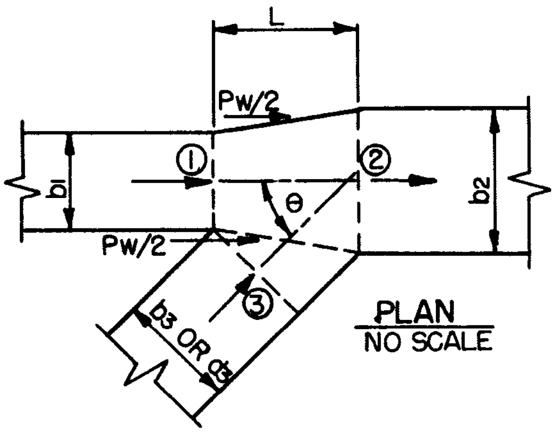
C. Derivation of Formula

1. Rectangular Section

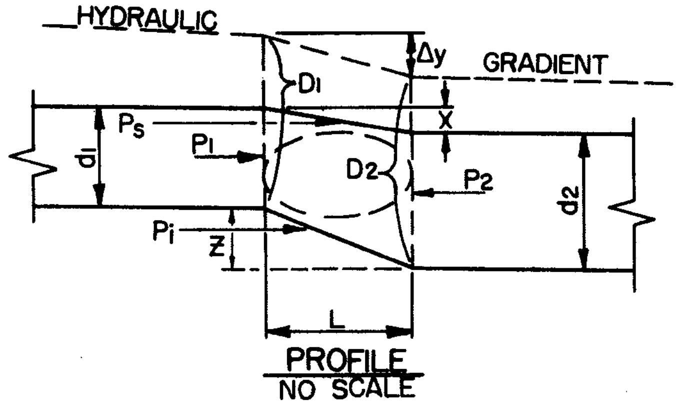
Derivation of Equation (4), by D. Thompson.

Rectangular Box with expansion, friction ignored.

C. Derivation of Formula (Continued)



PLAN
NO SCALE



PROFILE
NO SCALE

$$P_1 - P_2 + P_i - P_s + P_w = M_2 - M_1 - M_3 \cos\theta$$

$$X + d_2 = d_1 + Z \qquad X = Z + d_1 - d_2$$

$$\Delta y + D_2 = D_1 + Z \qquad \Delta y = Z + D_1 - D_2$$

$$P_1 = b_1 d_1 (D_1 - \tfrac{1}{2} d_1)$$

$$P_2 = b_2 d_2 (D_2 - \tfrac{1}{2} d_2)$$

$$P_i = {}^1/_6 Z \left[b_1 D_1 + 4(\tfrac{1}{2})(b_1 + b_2)(\tfrac{1}{2})(D_1 + D_2) + b_2 D_2\right]$$

$$P_s = {}^1/_6 X \left[b_1 (D_1 - d_1) + 4(\tfrac{1}{2})(b_1 + b_2)(\tfrac{1}{2})(D_1 - d_1 + D_2 - d_2) + b_2 (D_2 - d_2)\right]$$

$$P_w = {}^1/_6 (\tfrac{1}{2})(b_2 - b_1)\left[\tfrac{1}{2}(D_1 + D_1 - d_1) d_1 + 4(\tfrac{1}{2})(d_1 + d_2)(\tfrac{1}{2})\left(\tfrac{1}{2}(2D_1 - d_1) + \tfrac{1}{2}(2D_2 - d_2)\right) + \tfrac{1}{2}(D_2 + D_2 - d_2) d_2\right]^2$$

$$\Sigma P = \Delta y (\text{Average Area})$$

$$\Sigma P = P_1 - P_2 + P_i - P_s + P_w$$

$$P_1 = b_1 d_1 D_1 - \tfrac{1}{2}(b_1 d_1^2) = {}^1/_6 (6 b_1 d_1 D_1 - 3 b_1 d_1^2)$$

C. Derivation of Formula (Continued)

$P_2 = b_2d_2D_2-\frac{1}{2}(b_2d_2^2) = {}^1/_6(6b_2d_2D_2-3b_2d_2^2)$

$P_i = {}^1/_6Z(b_1D_1+b_1D_1+b_1D_2+b_2D_1+b_2D_2+b_2D_2)$

$= {}^1/_6Z(2b_1D_1+b_1D_2+b_2D_1+2b_2D_2)$

$P_s = {}^1/_6(Z+d_1-d_2)(b_1D_1-b_1d_1+b_1D_1-b_1d_1+b_1D_2-b_1d_2$

$+b_2D_1-b_2d_1+b_2D_2-b_2d_2+b_2D_2-b_2d_2)$

$= {}^1/_6(Z+d_1-d_2)(2b_1D_1-2b_1d_1+b_1D_2-b_1d_2+b_2D_1-b_2d_1$

$+2b_2D_2-2b_2d_2)$

$= {}^1/_6(2b_1D_1Z-2b_1d_1Z+b_1D_2Z-b_1d_2Z+b_2D_1Z-b_2d_1Z$

$+2b_2D_2Z-2b_2d_2Z+2b_1d_1D_1-2b_1d_1^2+b_1d_1D_2-b_1d_1d_2$

$+b_2d_1D_1-b_2d_1^2+2b_2d_1D_2-2b_2d_1d_2-2b_1d_2D_1+2b_1d_1d_2$

$-b_1d_2D_2+b_1d_2^2-b_2D_1d_2+b_2d_1d_2-2b_2d_2D_2+2b_2d_2^2)$

$= {}^1/_6(2b_1D_1Z-2b_1d_1Z+b_1D_2Z-b_1d_2Z+b_2D_1Z-b_2d_1Z$

$+2b_2D_2Z-2b_2d_2Z+2b_1d_1D_1-2b_1d_1^2+b_1d_1D_2+b_1d_1d_2$

$+b_2d_1D_1-b_2d_1^2+2b_2d_1D_2-b_2d_1d_2-2b_1d_2D_1-b_1d_2D_2$

$+b_1d_2^2-b_2D_1d_2-2b_2d_2D_2+2b_2d_2^2)$

$P_w = {}^1/_6(b_2-b_1)(D_1d_1-\frac{1}{2}d_1^2+d_1D_1-\frac{1}{2}d_1^2+d_1D_2-\frac{1}{2}d_1d_2$

$+d_2D_1-\frac{1}{2}d_1d_2+d_2D_2-\frac{1}{2}d_2^2+D_2d_2-\frac{1}{2}d_2^2)$

$= {}^1/_6(b_2-b_1)(2d_1D_1-d_1^2+d_1D_2-d_1d_2+d_2D_1+2d_2D_2-d_2^2)$

C. Derivation of Formula (Continued)

$$= {}^1/_6(2b_2d_1D_1-b_2d_1{}^2+b_2d_1D_2-b_2d_1d_2+b_2d_2D_1{}^2+b_2d_2D_2$$
$$-b_2d_2{}^2-2b_1d_1D_1+b_1d_1{}^2-b_1d_1D_2+b_1d_1d_2-b_1d_2D_1$$
$$-2b_1d_2D_2+b_1d_2{}^2)$$

$$\Sigma P = {}^1/_6(6b_1d_1D_1-3b_1d_1{}^2-6b_2d_2D_2+3b_2d_2{}^2+2b_1D_1Z$$
$$+b_1D_2Z+b_2D_1Z+2b_2D_2Z-2b_1D_1Z-b_1D_2Z-b_2D_1Z-2b_2D_2Z$$
$$+2b_1d_1Z+b_1d_2Z+b_2d_1Z+2b_2d_2Z-2b_1d_1D_1+2b_1d_1{}^2$$
$$-b_1d_1D_2-b_1d_1d_2-b_2d_1D_1+b_2d_1{}^2-2b_2d_1D_2+b_2d_1d_2$$
$$+2b_1d_2D_1+b_1d_2D_2-b_1d_2{}^2+b_2D_1d_2+2b_2d_2D_2-2b_2d_2{}^2$$
$$+2b_2d_1D_1-b_2d_1{}^2+b_2d_1D_2-b_2d_1d_2+b_2d_2D_1+2b_2d_2D_2$$
$$-b_2d_2{}^2-2b_1d_1D_1+b_1d_1{}^2-b_1d_1D_2+b_1d_1d_2-b_1d_2D_1$$
$$-2b_1d_2D_2+b_1d_2{}^2)$$

$$\Sigma P = {}^1/_6(2b_1d_1D_1-2b_1d_1D_2-2b_2d_2D_2+b_2d_1D_1-b_2d_1D_2$$
$$+b_1d_2D_1-b_1d_2D_2+2b_2d_2D_1+2b_1d_1Z+b_1d_2Z+b_2d_1Z$$
$$+2b_2d_2Z)$$

$$\Delta y(\text{AVERAGE AREA}) = (Z+D_1-D_2)\,{}^1/_6\Big[\,b_1d_1$$
$$+4(\tfrac{1}{2})(b_1+b_2)\tfrac{1}{2}(d_1+d_2)+b_2d_2\,\Big]$$
$$= {}^1/_6(b_1d_1+b_1d_1+b_1d_2+b_2d_1+b_2d_2$$
$$+b_2d_2)(Z+D_1-D_2)$$
$$= {}^1/_6(2b_1d_1+b_1d_2+b_2d_1+2b_2d_2)(Z$$
$$+D_1-D_2)$$

C. Derivation of Formula (Continued)

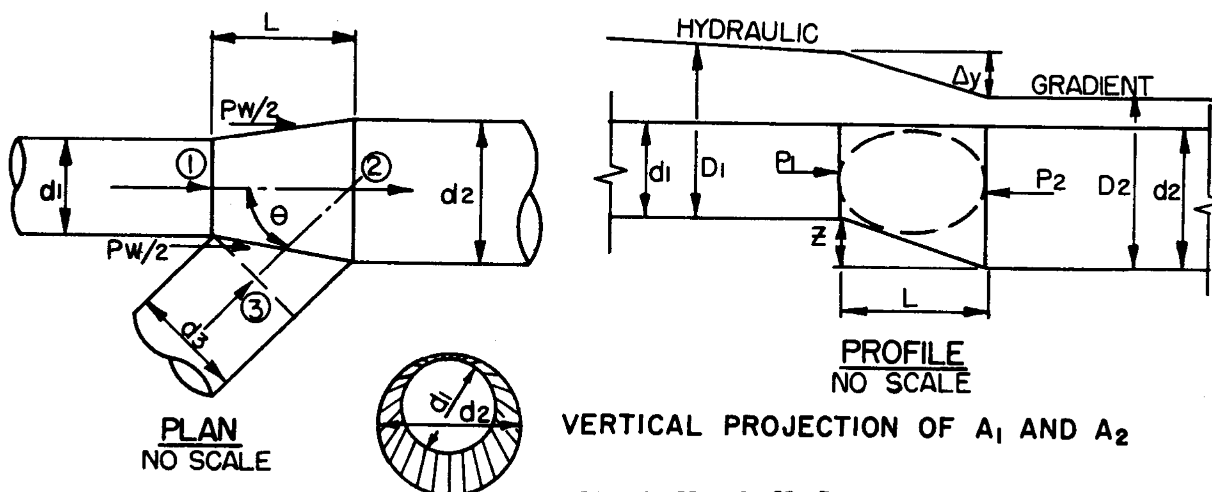
$$\Delta y(\text{AVERAGE AREA}) = {}^1/_6(2b_1d_1D_1-2b_1d_1D_2-2b_2d_2D_2 +2b_2d_2D_1+b_1d_2D_1+b_2d_1D_1-b_1d_2D_2 -b_2d_1D_2+2b_1d_1Z+b_1d_2Z+b_2d_1Z +2b_2d_2Z)$$

$$\Sigma P = \Delta y(\text{AVERAGE AREA}) = \Sigma M = M_2-M_1-M_3\text{Cos}\,\Theta$$

2. Circular Section

Derivation of Equation (4), by D. Thompson.

Circular conduit with expansion, friction ignored.



VERTICAL PROJECTION OF A_1 AND A_2

$$P_1+P_w+P_i-P_2 = \frac{Q_2V_2-Q_1V_1-Q_3V_3\text{Cos}\,\Theta}{g}$$

$$P_1 = A_1\bar{y}_1 = \pi/_4 d_1{}^2(D_1-\tfrac{1}{2}d_1) = \pi/_8(2D_1d_1{}^2-d_1{}^3)$$

$$P_2 = A_2\bar{y}_2 = \pi/_4 d_2{}^2(D_2-\tfrac{1}{2}d_2) = \pi/_8(2D_2d_2{}^2-d_2{}^3)$$

$$P_i = 0$$

$$P_w = A_w\bar{y}_w$$

$$A_w = \text{VERTICAL PROJECTION OF WALL} = A_2-A_1$$

C. Derivation of Formula (Continued)

$\bar{y}_w$ = C.G. of VERTICAL PROJECTION OF TRANSITION PLUS AVERAGE DISTANCE OF H.G. ABOVE TRANSITION SOFFIT.

$$A_w = \pi/4(d_2{}^2-d_1{}^2)$$

$$\text{C.G. FROM SOFFIT} = \frac{\pi/4(d_2{}^2)(\tfrac{1}{2}d_2)-\pi/4(d_1{}^2)(\tfrac{1}{2}d_1)}{\pi/4(d_2{}^2-d_1{}^2)}$$

$$= \frac{d_2{}^3-d_1{}^3}{2(d_2{}^2-d_1{}^3)}$$

$$\bar{y}_w = \frac{d_2{}^3-d_1{}^3}{2(d_2{}^2-d_1{}^2)} + \tfrac{1}{2}(D_1-d_1)+\tfrac{1}{2}(D_2-d_2)$$

$$P_w = A_w\bar{y}_w$$

$$= \pi/4(d_2{}^2-d_1{}^2)\left(\frac{d_2{}^3-d_1{}^3}{2(d_2{}^2-d_1{}^2)} + \frac{D_1+D_2-d_1-d_2}{2}\right)$$

$$\Sigma P = P_1+P_w-P_2$$

$$= \pi/8(2D_1d_1{}^2-d_1{}^3+d_2{}^3-d_1{}^3+D_1d_2{}^2+D_2d_2{}^2-D_1d_1{}^2-D_2d_1{}^2$$
$$-d_1d_2{}^2-d_1{}^3-d_2{}^3+d_2d_1{}^2-2D_2d_2{}^2+d_2{}^3)$$

$$\Delta y = (D_1-d_1)-(D_2-d_2);$$

$$\text{AVERAGE AREA} = \tfrac{1}{2}(A_1+A_2) = \pi/8(d_1{}^2+d_2{}^2)$$

$$\Delta y(\text{AVERAGE AREA}) = \pi/8(D_1d_1{}^2-D_2d_1{}^2-d_1{}^3+d_2d_1{}^2+D_1d_2{}^2$$
$$-D_2d_2{}^2-d_1d_2{}^2+d_2{}^3)$$

BY INSPECTION: Δy(AVERAGE AREA) = ΣP = Σ MOMENTUMS

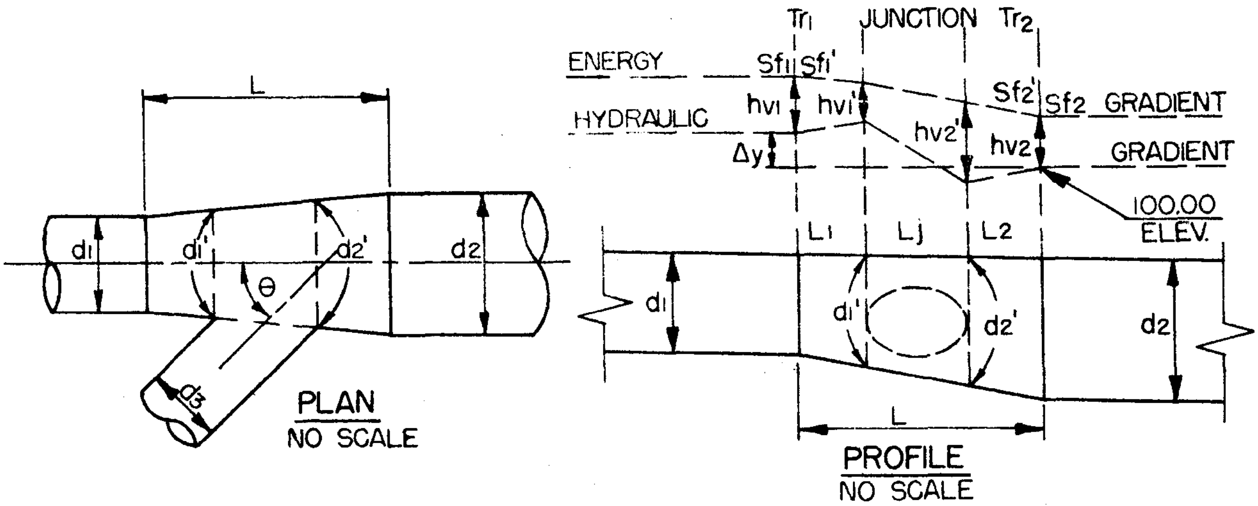
C. Derivation of Formula (Continued)

$$\Delta y(\text{AVERAGE AREA}) = \frac{Q_2V_2 - Q_1V_1 - Q_3V_3\cos\theta}{g} \qquad (4)$$

$$= \frac{Q_2{}^2}{A_2g} - \frac{Q_1{}^2}{A_1g} - \frac{Q_3{}^2\cos\theta}{A_3g}$$

D. Example: Circular Section

Determine hydraulic and energy gradient across junction and across structure.



Pipe 1	Pipe 2	Pipe 3	
d_1=4.5'(54")	d_2=5.5'(66")	d_3=3.5'(42")	θ=45°
Q_1=223 cfs	Q_2=380 cfs	Q_3=157 cfs	n=0.013
A_1=15.9 sq.ft.	A_2=23.8 sq.ft.	A_3=9.6 sq.ft.	L=8.8 ft.
V_1=14.0 fps	V_2=16.0 fps	V_3=16.4 fps	L_1=2.6 ft.
h_{v1}=3.04 ft.	h_{v2}=3.98 ft.	h_{v3}=4.17 ft.	L_j=4.7 ft.
S_{f1}=0.0128	S_{f2}=0.0128	S_{f3}=0.0246	L_2=1.5 ft.
d_1'=4.8'(57.6")	d_2'=5.3'(63.6")		
A_1'=18.09 sq.ft.	A_2'=22.1 sq.ft.		
V_1'=12.3 fps	V_2'=17.3 fps		
h_{v1}'=2.35 ft.	h_{v2}'=4.62 ft.		
S_{f1}'=0.0092	S_{f2}'=0.0156		

D. Example: Circular Section (Continued)

1. Transition Losses Considered

$$\Delta y_j(\text{AVERAGE AREA}) = \frac{Q_2V_2'-Q_1V_1'-Q_3V_3\cos\theta}{g}$$

$$+\tfrac{1}{2}(S_{f1}'+S_{f2}')L_j(\tfrac{1}{2})(A_1'+A_2')$$

$$\Delta y_j = \frac{Q_2V_2'-Q_1V_1'-Q_3V_3\cos\theta}{\tfrac{1}{2}(A_1'+A_2')g} +\tfrac{1}{2}(S_{f1}'+S_{f2}')L_j$$

$$\Delta y_j = \frac{380(17.3)-223(12.3)-157(16.4)(.707)}{\tfrac{1}{2}(18.1+22.1)32.2}$$

$$+\tfrac{1}{2}(0.0092+0.0156)$$

$$\Delta y_j = \underline{\underline{3.1584'}}$$

$$h_j = \Delta y_j+h_{v1}'-h_{v2}' = 3.158+2.35-4.62 = \underline{\underline{0.888'}}$$

$$h_{tr1} = 0.0032(V_1-V_1')^2+\tfrac{1}{2}(S_{f1}+S_{f1}')L_1$$

$$= 0.0032(14.0-12.3)^2+\tfrac{1}{2}(0.0128+0.0092)(2.6)$$

$$= \underline{\underline{0.0378}}$$

$$h_{tr2} = 0.0032(V_2'-V_2)^2+\tfrac{1}{2}(S_{f2}+S_{f2}')L_2$$

$$= 0.0032(17.3-16.0)^2+\tfrac{1}{2}(0.0128+0.0156)(1.5)$$

$$= \underline{\underline{0.0267}}$$

Computing Hydraulic and Energy Grade

		H.G.	E.G.
Assume H.G. at Pipe 2	= 100.00	100.000	
E.G.	= 100.00+3.980		103.980
AT d_2': E.G.	= 103.980+0.0267		104.007
H.G.	= 104.007-4.620	99.387	
AT d_1': H.G.	= 99.387+3.158	102.545	
E.G.	= 102.545+2.350		104.895
AT d_1 : E.G.	= 104.895+0.0378		104.933
H.G.	= 104.933-3.040	101.893	

D. Example: Circular Section (Continued)

2. Transition Losses Ignored

Using d_1 and d_2

$$\Delta y = \frac{Q_2V_2-Q_1V_1-Q_3V_3\text{Cos}\,\Theta}{\frac{1}{2}(A_1+A_2)g} + \tfrac{1}{2}(S_{f1}+S_{f2})L$$

$$\Delta y = \frac{380(16.0)-223(14.0)-157(16.4)0.707}{\frac{1}{2}(15.9+23.8)32.2}$$

$$+\tfrac{1}{2}(0.0128+0.0128)(8.8)$$

$$\Delta y = \frac{1138}{32.2(19.85)} + 8.8(0.0128) = \underline{\underline{1.8930}}$$

		H.G.	E.G.
AT d_2 :	H.G. = 100.00	100.000	
	E.G. = 100.00+3.980		103.980
AT d_1 :	H.G. = 100.00+1.893	101.893	
	E.G. = 101.893+3.040		104.933

Difference: $104.933-104.933 = \underline{\underline{0.000'}}$

AT d_1':

Lateral H.G. = $H.G.(1)+h_{v1}-h_{v1}'-S_{f1}'L_1$

Lateral H.G. = $101.893+3.040-2.350-0.024 = 102.559$

Difference: $102.559-102.545 = \underline{\underline{0.014'}}$

Values from Methods 1 and 2 are approximately equal. Method 2 using the average end areas should be used in determining the junction loss. The transition losses are small enough to be ignored.