I FOREWORD AND ACKNOWLEDGMENT

The general formula for the hydraulic analysis of junctions which has been used in this monograph was derived by Donald Thompson, Chief Engineer of Design, Bureau of Engineering, City of Los Angeles. The formula is based on the well-known pressure plus momentum theory which states that the change in pressure through a junction is equal to the change in momentum. The application of the formula to actual design problems, the determination of control points, and the graphical solutions for conditions where a direct solution was not possible were prepared by Irving R. Cole, Division Engineer, Storm Drain Design Division.

Valuable assistance and advice have been given by Floyd J. Doran, Deputy City Engineer. Numerous model tests conducted over a period of several years at the Experimental Hydraulic Research Laboratory of the Bureau of Engineering have confirmed the accuracy of the Thompson formula and the pressure plus momentum theory.
<table>
<thead>
<tr>
<th>INDEX</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I Foreword and Acknowledgment</td>
<td>1</td>
</tr>
<tr>
<td>II Introduction</td>
<td>5-8</td>
</tr>
<tr>
<td>A. Purpose and Objectives</td>
<td>5</td>
</tr>
<tr>
<td>B. Notation</td>
<td>5-7</td>
</tr>
<tr>
<td>C. General Formula</td>
<td>7-8</td>
</tr>
<tr>
<td>III Open Channel Flow</td>
<td>8-10</td>
</tr>
<tr>
<td>A. Criteria for Junction</td>
<td>8</td>
</tr>
<tr>
<td>B. General Formula</td>
<td>8-9</td>
</tr>
<tr>
<td>C. Control Points</td>
<td>9-10</td>
</tr>
<tr>
<td>1. Subcritical Flow</td>
<td>9</td>
</tr>
<tr>
<td>2. Supercritical Flow</td>
<td>9-10</td>
</tr>
<tr>
<td>D. Derivation of Formula</td>
<td>10-12</td>
</tr>
<tr>
<td>1. Rectangular Section</td>
<td>10-11</td>
</tr>
<tr>
<td>2. Circular Section</td>
<td>11-12</td>
</tr>
<tr>
<td>E. Outline of Examples</td>
<td>12-14</td>
</tr>
<tr>
<td>1. Rectangular Section-Subcritical Flow</td>
<td>12-13</td>
</tr>
<tr>
<td>2. Rectangular Section-Supercritical Flow</td>
<td>13</td>
</tr>
<tr>
<td>3. Circular Section-Subcritical Flow</td>
<td>13</td>
</tr>
<tr>
<td>4. Circular Section-Supercritical Flow</td>
<td>13-14</td>
</tr>
<tr>
<td>F. Examples</td>
<td>14-27</td>
</tr>
<tr>
<td>1. Rectangular Section-Subcritical Flow</td>
<td>14-16</td>
</tr>
<tr>
<td>2. Rectangular Section-Supercritical Flow</td>
<td>16-21</td>
</tr>
<tr>
<td>3. Circular Section-Subcritical Flow</td>
<td>21-23</td>
</tr>
<tr>
<td>4. Circular Section-Supercritical Flow</td>
<td>23-27</td>
</tr>
<tr>
<td>IV Pressure Flow</td>
<td>27-37</td>
</tr>
<tr>
<td>A. Criteria for Junction</td>
<td>27-28</td>
</tr>
<tr>
<td>B. General Conditions</td>
<td>28-29</td>
</tr>
<tr>
<td>C. Derivation of Formula</td>
<td>29</td>
</tr>
<tr>
<td>1. Rectangular Section</td>
<td>29-33</td>
</tr>
<tr>
<td>2. Circular Section</td>
<td>33-35</td>
</tr>
</tbody>
</table>
INDEX (Continued)

D. Example: Circular Section

1. Transition Losses Considered ............................................. 36
2. Transition Losses Ignored .................................................. 37
II INTRODUCTION

A. Purpose and Objectives

Junctions in conduits can cause major losses in both the energy grade and the hydraulic grade across the junction. If these losses are not included in the hydraulic design, the capacity of the conduit may be seriously restricted. The pressure plus momentum theory, which equates the summation of all pressures acting at the junction with the summation of the momentums, affords a rational method of analyzing the hydraulic losses at a junction. The pressures which must be evaluated are (1) upstream end of the junction, (2) downstream end of the junction, (3) wall pressures, (4) invert pressure, and (5) soffit pressure. Formulas for the above pressures, derived from principles of hydrostatics, are extremely complicated, difficult if not impossible to remember, and, because of their complexities, may result in frequent errors. The general formula used in this monograph makes it unnecessary to evaluate individual pressures. It can be shown (see below) that, regardless of the shape of the conduit, the summation of all pressures acting at the junction, ignoring friction, is equal to the average cross-sectional area through the junction, multiplied by the change in the hydraulic gradient through the junction.

The following discussion, together with the sample problems and their solutions, illustrate the use of the general formula in determining the hydraulic changes at a junction. The discussion includes (1) the derivation of the general formula for both rectangular and circular conduits under open flow and pressure flow conditions, (2) the determinations of the control points for subcritical and supercritical flow in open channels, and (3) the solution for the hydraulic grade of the lateral under pressure flow conditions.

B. Notation

(Unit Weight of Water Omitted)

Q
Discharge, cubic feet per second (cfs.).

A
Area of flow, square feet (ft²).

A_m
Mean Area, square feet (ft²).

b
Width of rectangular channel, feet.

d
Diameter, of circular conduit, feet.
B. Notation (Continued)

\( D \)  
Elevation of hydraulic gradient above invert, feet.  
Depth of flow, feet (open channel).

\( g \)  
Gravitational acceleration, 32.2 feet per second per second.

\( \Delta y \)  
Change in hydraulic gradient or water surface through the junction, feet. (Plus when increasing upstream.)

\( P \)  
Hydrostatic pressure, cubic feet.

\( P_i \)  
Longitudinal component of invert pressure, cubic feet.

\( P_s \)  
Longitudinal component of soffit pressure, cubic feet.

\( P_w \)  
Longitudinal component of wall pressure, cubic feet.

\( P_f \)  
Pressure loss due to friction, cubic feet (friction loss).

\( V \)  
Velocity, feet per second.

\( \theta \)  
Angle of convergence between the center line of the main line and the center line of the lateral, degrees.

\( L \)  
Length of junction, feet.

\( S \)  
Construction slope, feet per foot.

\( S_r \)  
Slope of the energy gradient, feet per foot.

\( M \)  
Momentum of a moving mass of water \( \left( \frac{QV}{g} \right) \), cubic feet.

\( n \)  
Mannings roughness coefficient.

\( Z \)  
Change in invert elevation across the junction, feet.

\( x \)  
Change in soffit elevation across the junction, feet.

\( \bar{y} \)  
Distance from hydraulic gradient to center of gravity of section, feet.

\( h_v \)  
Velocity head \( \left( \frac{V^2}{2g} \right) \), feet.

\( \phi \)  
Angle of divergence of transition, degrees.

\( \alpha \)  
Angle of invert slope of junction, degrees.
HYDRAULIC ANALYSIS OF JUNCTIONS

B. Notation (Continued)

h Energy loss, feet.
H.G. Hydraulic gradient.
E.G. Energy gradient.
t Transition.
T Top width of water surface, open channel.

Numerical subscript denotes position.
Subscript "j" denotes junction.
Subscript "c" denotes critical flow.
Subscript "n" denotes normal flow.
Subscript "tr" denotes transition.

C. General Formula

The net hydrostatic pressure at a junction equals the change in momentum through the junction plus friction.

GENERAL FORMULA WITH FRICTION INCLUDED
(UNIT WEIGHT OF WATER OMITTED)

\[ P_2 + M_2 = P_1 + M_1 + M_3 \cos \theta + P_w + P_1 - P_f \]  \( \text{(1)} \)

\[ P_1 + P_w + P_1 - P_2 = M_2 - M_1 = M_3 \cos \theta + P_f \]

Net hydrostatic pressure = \( \Sigma P = P_1 + P_w + P_1 - P_2 \)

\[ \Delta y (\text{AVERAGE AREA}) = P_1 + P_w + P_1 - P_2 \]  \( \text{(2)} \)

AVERAGE AREA = \( \frac{1}{6}(A_1 + 4A_m + A_2) \)

or for practical use \( \frac{1}{2}(A_1 + A_2) \)

\[ \frac{1}{2}(A_1 + A_2) \Delta y = M_2 - M_1 - M_3 \cos \theta + P_f \]

\[ Q_2 V_2 - Q_1 V_1 - Q_3 V_3 \cos \theta = \frac{L(S_1 + S_2)(A_1 + A_2)}{g} + \frac{L(S_1 + S_2)(A_1 + A_2)}{2} \]  \( \text{(3)} \)
HYDRAULIC ANALYSIS OF JUNCTIONS

C. General Formula (Continued)

Omitting friction, equation (3) is shown as:

\[ \frac{1}{2}(A_1+A_2)\Delta y = \frac{Q_2 V_2 - Q_1 V_1 - Q_3 V_3 \cos \theta}{g} \]  

\[ = \frac{Q_2^2}{A_2 g} - \frac{Q_1^2}{A_1 g} - \frac{Q_3^2 \cos \theta}{A_3 g} \]  

Equations 3, 4 and 5 are valid for all types of Prismaticoidal and Circular Channels and Conduits.

III OPEN CHANNEL FLOW

A. Criteria for Junction

Flow in channels and conduits with a free water surface is called open channel flow. The lateral inlet is considered to be either submerged or to have a water surface elevation approximately equal to the average water surface elevation through the junction. The hydraulic gradient or water surface elevations at Points 1 and 3 are identical.

B. General Formula

**\[ \Sigma P = \Sigma M \]**

\[ \frac{1}{2}(A_1+A_2)\Delta y = \frac{Q_2^2}{A_2 g} - \frac{Q_1^2}{A_1 g} - \frac{Q_3^2 \cos \theta}{A_3 g} \]  

- 8 -
HYDRAULIC ANALYSIS OF JUNCTIONS

B. General Formula (Continued)

\[ \Delta y = Z + D_1 - D_2 \]

\[ \frac{1}{2}(A_1 + A_2)(Z + D_1 - D_2) = \frac{Q_2^2}{A_2g} - \frac{Q_1^2}{A_1g} - \frac{Q_3^2 \cos \theta}{A_3g} \]  \hspace{1cm} (6)

C. Control Points

Flow may be either subcritical or supercritical.

1. Subcritical Flow

\[ D_2 = D_{2n} \]

2. Supercritical Flow

It is necessary to determine if a hydraulic jump will occur at the junction. The depth at the downstream end of the junction (Point 2) is set at critical depth, and the momentums calculated to determine if the incoming flows can maintain supercritical flow.

a. \[ M_{2c} < M_{1nc} + M_3 \cos \theta + \frac{1}{2}(A_1 + A_2) \Delta y \]

\[ D_1 = D_{1nc} \]

b. \[ M_{2c} > M_{1nc} + M_3 \cos \theta + \frac{1}{2}(A_1 + A_2) \Delta y \]

Hydraulic jump will occur

\[ D_2 = D_{2c} \]

Where the expanded structure is longer than the junction, it may be necessary to compute a drawdown water surface profile to determine the water depth and area at the point where the lateral enters the structure. Transition losses are negligible and can be ignored.
C. Control Points (Continued)

After determination of the change through the junction, backwater or drawdown calculations will have to be made to determine the water surface profile.

D. Derivation of Formula

1. Rectangular Section

Derivation of Equation (4) by D. Thompson.

Rectangular channel with expansion, friction ignored.

\[ P_1 + P_w + P_1 - P_2 = \frac{Q_2 V_2 - Q_1 V_1 - Q_3 V_3 \cos \theta}{g} \]

\[ P_1 = \frac{2}{3} (b_1 D_1^2) = \frac{3}{6} (b_1 D_1^2) \]

\[ P_w = \frac{1}{6} (D_1^2 + D_1 D_2 + D_2^2) (b_2 - b_1) \]

\[ P_w = \frac{1}{6} (b_2 D_1^2 + b_2 D_1 D_2 + b_2 D_2^2 - b_1 D_1 D_2 - b_1 D_2^2 - b_1 D_1^2) \]

\[ P_1 = \frac{1}{6} (2b_1 D_1 + b_1 D_2 + b_2 D_1 + 2b_2 D_2) Z \]

\[ P_2 = \frac{2}{3} (b_2 D_2^2) = \frac{3}{6} (b_2 D_2^2) \]

\[ \Sigma P = \frac{3}{6} (b_1 D_1^2) + \frac{1}{6} (2b_1 D_1 + b_1 D_2 + b_2 D_1 + 2b_2 D_2) Z \]

\[ + \frac{1}{6} (b_2 D_1^2 + b_2 D_1 D_2 + b_2 D_2^2 - b_1 D_1 D_2 - b_1 D_1^2 - b_1 D_2^2) \]

\[ - \frac{3}{6} (b_2 D_2^2) \]
D. Derivation of Formula

\[ \Delta y = Z + D_1 - D_2 \]

Average Area = \( \frac{1}{6} (2b_1D_1 + b_1D_2 + b_2D_1 + 2b_2D_2) \)

\[ \Delta y (\text{Average Area}) = \frac{1}{6} (2b_1D_1 + b_1D_2 + b_2D_1 + 2b_2D_2)Z \]

\[ + \frac{1}{6} (2b_1D_1^2 - b_1D_1D_2 + b_2D_1^2 + b_2D_1D_2 - b_1D_2^2 + 2b_2D_2^2) \]

\[ \Sigma P = P_1 + P_w + P_1 - P_2 \]

By Inspection: \( \Delta y (\text{Average Area}) = \Sigma P = \Sigma \text{Moments} \)

\[ \Delta y (\text{Average Area}) = \frac{Q_2V_2 - Q_1V_1 - Q_3V_3 \cos \theta}{g} \tag{4} \]

2. Circular Section

Derivation of Equation (4) by D. Thompson.

Circular conduit with expansion, friction ignored.
D. Derivation of Formula (Continued)

\[ P_2 = A_2 \bar{y}_2 \quad P_1 = 0 \]

\[ A_w = A_2 + \frac{1}{3} (T_1 + T_2) \Delta y - A_1 \]

\[ \bar{y}_w = \bar{y}_{xx} - \frac{\Delta y}{2} \]

\[ \bar{y}_w = \frac{A_2 (\bar{y}_2 + \Delta y) + \frac{1}{3} T_1 \Delta y^2 + \frac{1}{2} (T_2 - T_1) \left( \frac{1}{8} \Delta y \right)^2}{A_2 + \frac{1}{3} (T_1 + T_2) \Delta y - A_1} \]

\[ - \frac{A_1 \bar{y}_1}{A_2 + \frac{1}{3} (T_1 + T_2) \Delta y - A_1} - \frac{\Delta y}{2} \]

\[ \bar{y}_w = A_2 (\bar{y}_2 + \Delta y) + \frac{1}{3} \Delta y^2 \left( \frac{1}{2} T_1 + T_2 \right) - A_1 \bar{y}_1 \]

\[ - \frac{\Delta y}{2} \left[ A_2 + \frac{1}{3} (T_1 + T_2) \Delta y - A_1 \right] \]

\[ P_w = A_w \bar{y}_w = A_2 (\bar{y}_2 + \Delta y) - A_1 \bar{y}_1 + \frac{1}{3} \Delta y^2 \left( \frac{1}{2} T_1 + T_2 \right) \]

\[ 1/12 \Delta y^2 (T_2 - T_1) \text{ will be small and can be omitted.} \]

\[ \Sigma P = P_1 + P_w - P_2 = A_1 \bar{y}_1 + A_2 \bar{y}_2 - A_1 \bar{y}_1 + \frac{1}{2} (A_1 + A_2) \Delta y - A_2 \bar{y}_2 \]

\[ \Sigma P = \frac{1}{2} (A_1 + A_2) \Delta y = \Delta y \text{(Average Area)} = \Sigma \text{Momentums} \]

\[ \Delta y \text{(AVERAGE AREA)} = \frac{Q_2 V_2 - Q_1 V_1 - Q_3 V_3 \cos \theta}{g} \quad (4) \]

E. Outline of Examples

1. Rectangular Section - Subcritical Flow

Case A: Determine \( Z \) so that \( D_1 = D_{1n} \)
E. Outline of Examples (Continued)

Case B: Determine D₁ when Z = 0

2. Rectangular Section - Supercritical Flow

Case C: Determine Z when D₁ = D₁₀ so that a hydraulic jump cannot form.

\[ M₂ \leq M₁₀ + M₃ \cos \theta + \frac{1}{2}(A₁ + A₂)Δy \]

\[ M₂ = M₂C \]

\[ M₂ = M₂n \]

Case D: Determine D₁ when Z = 0 and D₂ = D₂₀

\[ M₂₀ > M₁₀ + M₃ \cos \theta + \frac{1}{2}(A₁ + A₂)Δy \]

Case E: Determine D₂ when Z = 0 and D₁ = D₁₀

\[ M₂₀ < M₁₀ + M₃ \cos \theta + \frac{1}{2}(A₁ + A₂)Δy \]

3. Circular Section - Subcritical Flow

Case F: Determine D₁ when Z = d₂ - d₁ = 0.5'

Case G: Determine Z so that D₁ = \( \frac{3}{4}d₁ \)

and D₂ = \( \frac{3}{4}d₂ \)

4. Circular Section - Supercritical Flow

Case H: Determine D₁ when Z = d₂ - d₁ = 0.5'

and D₂ = D₂₀

\[ M₂₀ > M₁₀ + M₃ \cos \theta + \frac{1}{2}(A₁ + A₂)Δy \]

Case I: Determine Z so that a hydraulic jump cannot form.

- 13 -
E. Outline of Examples (Continued)

\[ M_2 \leq M_1 n + M_3 \cos \theta + \frac{1}{2} (A_1 + A_2) \Delta y \]

\[ M_2 = M_2 c \]

\[ M_2 = M_2 n \]

Case J: Determine \( D_2 \) when \( D_1 = D_{1n} \) and \( Z = 0.75' \)

\[ M_{2c} < M_1 n + M_3 \cos \theta + \frac{1}{2} (A_1 + A_2) \Delta y \]

F. Examples

1. Rectangular Section - Subcritical Flow

Case A: When flow is subcritical both upstream and downstream, \( D_2 = D_{2n} \); SET \( D_1 = D_{1n} \) and solve for value of \( Z \) required to maintain normal flow.

GIVEN: \( Q_1 = 11,015 \) cfs \( Q_2 = 11,450 \) cfs \( Q_3 = 435 \) cfs

\( b_1 = 38.67 \) ft. \( b_2 = 38.67 \) ft. \( d_3 = 93 \) in.

\( S_1 = 0.0022 \) \( S_2 = 0.0020 \) \( A_3 = 47 \) sq.ft.

\( D_{1n} = 14.1 \) ft. \( D_{2n} = 15.0 \) ft. \( \theta = 30^\circ \)

\( A_{1n} = 545 \) sq.ft. \( A_{2n} = 582 \) sq.ft. \( n = 0.014 \)
F. Examples (Continued)

FORMULA AND SOLUTION:

\[ \Sigma P = \Sigma M \]

\[ \frac{1}{2}(A_1 + A_2)(Z + D_1 - D_2) = \frac{Q_2^2}{A_2g} - \frac{Q_1^2}{A_1g} - \frac{Q_3^2 \cos \theta}{A_3g} \]

\[ \frac{1}{2}(545 + 582)(Z + 14.1 - 15.0) = \frac{(11.450)^2}{582(32.2)} - \frac{(11.015)^2}{545(32.2)} - \frac{(435)^2(0.866)}{47(32.2)} \]

563.5(Z - 0.9) = 6990 - 6940 - 108

\[ Z = 0.9 - \frac{58}{563.5} \]

\[ Z = 0.80 \text{ ft.} \]

\[ \Delta y = Z + D_1 - D_2 \]

\[ = 0.80 + 14.1 - 15.0 \]

\[ = -0.10 \text{ ft.} \]

Case B: When \( Z \) is limited or is set a certain value; \( D_2 = D_2n \), DETERMINE \( D_1 \) when \( Z = 0 \).
F. Examples (Continued)

**GIVEN:**

- \(Q_1 = 11,015 \text{ cfs}\)
- \(Q_2 = 11,450 \text{ cfs}\)
- \(Q_3 = 435 \text{ cfs}\)
- \(b_1 = 38.67 \text{ ft}\.
- \(b_2 = 38.67 \text{ ft}\.
- \(d_3 = 93 \text{ in.}\)
- \(S_1 = 0.0022\)
- \(S_2 = 0.0020\)
- \(A_3 = 47 \text{ sq ft.}\)
- \(D_{1n} = 14.1 \text{ ft.}\)
- \(D_{2n} = 15.0 \text{ ft.}\)
- \(\theta = 30^\circ\)
- \(A_{2n} = 582 \text{ sq ft.}\)
- \(n = 0.014\)

**FORMULA:**

\[
\frac{\Sigma P}{\Sigma M} = \frac{Q_2^2}{A_2 g} - \frac{Q_1^2}{A_1 g} - \frac{Q_3^2 \cos \theta}{A_3 g}
\]

\[
\frac{\Sigma (A_1 + A_2)(Z + D_1 - D_2)}{\Sigma (A_1 + 582)(Z + D_1 - 15.0)} = 6990 - \frac{3,775,000}{A_1} - 108
\]

\[
\frac{\Sigma (A_1 + 582)(D_1 - 15.0)}{\Sigma (A_1 + 582)(D_1 - 15.0)} = 6882 - \frac{3,775,000}{A_1}
\]

**SOLUTION:**

<table>
<thead>
<tr>
<th>(D_1)</th>
<th>(A_1)</th>
<th>(\Sigma P)</th>
<th>(\Sigma M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.0</td>
<td>620</td>
<td>601</td>
<td>790</td>
</tr>
<tr>
<td>16.5</td>
<td>639</td>
<td>918</td>
<td>970</td>
</tr>
<tr>
<td>16.75</td>
<td>648</td>
<td>1075</td>
<td>1050</td>
</tr>
<tr>
<td>17.0</td>
<td>657</td>
<td>1240</td>
<td>1140</td>
</tr>
</tbody>
</table>

**PLOT** \(D_1\) **VERSUS** \(\Sigma P\) and \(\Sigma M\):

\(D_1 = 16.7 \text{ ft.}\)

\(\Delta y = Z + D_1 - D_2 = 0 + 16.7 - 15.0 = 1.7 \text{ ft.}\)

To complete the computation, determine water surface profile from \(D_1 = 16.7 \text{ ft.}\) to \(D_{1n} = 14.1 \text{ ft.}\)

---

2. **Rectangular Section - Supercritical Flow**

**Case C:** Supercritical flow upstream and downstream; solve for value of \(Z\) so that a hydraulic jump
F. Examples (Continued)

cannot occur at the junction. SET $D_1 = D_{1n}$, MAXIMUM $D_2 = D_{2c}$.

GIVEN: $Q_1 = 11,015$ cfs $Q_2 = 11,450$ cfs $Q_3 = 435$ cfs
$b_1 = 38.67$ ft. $b_2 = 38.67$ ft. $d_3 = 93$ in.
$S_1 = 0.00357$ $S_2 = 0.00357$ $A_3 = 47$ sq.ft.
$D_{1n} = 11.90$ ft. $D_{2c} = 13.98$ ft. $\theta = 30^\circ$
$A_{1n} = 460$ sq.ft. $A_{2c} = 540$ sq.ft. $n = 0.014$

$D_{2n} = 12.22$ ft.

FORMULA AND SOLUTION:

$M_{2c} \leq M_{1n} + M_3 \cos \theta + \frac{1}{2}(A_{1n} + A_{2c})(Z + D_{1n} - D_{2c})$

\[
\frac{(11,450)^2}{540(32.2)} = \frac{(11,015)^2}{460(32.2)} + \frac{(435)^2(0.866)}{47(32.2)} + \frac{1}{2}(460 + 540)(Z + 11.90 - 13.98)
\]

\[
7540 \leq 8210 + 108 + (500)(Z - 2.08)
\]

$Z \geq 0.52$ ft.

ALTERNATE SOLUTION: CONSIDER $D_2 = D_{2n} = 12.22$ ft.

$A_{2n} = 473$ sq.ft.

$M_{2n} = M_{1n} + M_3 \cos \theta + \frac{1}{2}(A_{1n} + A_{2n})(Z + D_{1n} - D_{2n})$
F. Examples (Continued)

\[ 8620 = 8210 + 108 + 466z - 149 \]

\[ z = 0.97 \text{ ft.} \]

\[ \Delta y = z + D_{1n} - D_{2n} = 0.97 + 11.90 - 12.22 = 0.65 \text{ ft.} \]

Case D: Conditions are identical to those for Case C, except that \( z = 0 \); DETERMINE \( D_1 \) and \( D_2 \).

**PROFILE**

**NOTE:**

NO SCALE

CONJ. = CONJUGATE

**DETERMINE POINT OF CONTROL:**

\[ M_{2c} = 7540 \]

\[ M_{1n} + M_3 \cos \theta + \frac{1}{2}(A_{1n} + A_{2c})(z + D_1 - D_2) = 8210 + 108 + 500(-2.08) = 7278 \]

\( M_{2c} \) is larger \((7540 > 7278)\)

A HYDRAULIC JUMP WILL FORM UPSTREAM OF THE JUNCTION.

\( D_2 - D_{2c} = 13.98 \text{ ft.} \); DETERMINE \( D_1 \).

**FORMULA:**

\[ \Sigma P = \Sigma M \]

\[ \frac{1}{2}(A_1 + A_2)(z + D_1 - D_2) = \frac{Q_2^2}{A_2g} - \frac{Q_1^2}{A_1g} - \frac{Q_3^2 \cos \theta}{A_3g} \]
HYDRAULIC ANALYSIS OF JUNCTIONS

F. Examples (Continued)

\[ \frac{1}{2}(A_1+540)(0+D_1-13.98) = 7540 - \frac{3.775.000}{A_1} - 108 \]

SOLUTION:

<table>
<thead>
<tr>
<th>D_1</th>
<th>A_1</th>
<th>ΣP</th>
<th>ΣM</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.0</td>
<td>619</td>
<td>1171</td>
<td>1330</td>
</tr>
<tr>
<td>16.5</td>
<td>638</td>
<td>1482</td>
<td>1520</td>
</tr>
<tr>
<td>17.0</td>
<td>659</td>
<td>1808</td>
<td>1690</td>
</tr>
</tbody>
</table>

PLOT D_1 VERSUS ΣP and ΣM:

D_1 = 16.7 ft.

Δy = Z+D_1-D_2 = 0+16.7-13.98 = 2.72 ft.

D_{1n} = 11.90 ft., D_{2n} = 12.22 ft.

\[ D_{1\text{Conj.}} = \frac{1}{2}D_{1n} + \sqrt{\frac{2Q_1^2(D_{1n})}{A_{1n}^2 \varepsilon}} = 15.55 \text{ ft.} \]

DETERMINE WATER SURFACE PROFILE FOR:

(a) D_1 (16.7 ft.) TO D_{1\text{Conj.}} (15.55 ft.)

(b) D_{2c} (13.98 ft.) TO D_{2n} (12.22 ft.)

Case E: Supercritical flow upstream and downstream, Z=0.
F. Examples (Continued)

GIVEN: \( Q_1 = 11,015 \text{ cfs} \) \( Q_2 = 11,450 \text{ cfs} \) \( Q_3 = 435 \text{ cfs} \)
\( b_1 = 38.67 \text{ ft.} \) \( b_2 = 38.67 \text{ ft.} \) \( d_3 = 93 \text{ in.} \)
\( S_1 = 0.00582 \) \( S_2 = 0.00582 \) \( A_3 = 47 \text{ sq. ft.} \)
\( D_{1n} = 10.0 \text{ ft.} \) \( D_{2c} = 13.98 \text{ ft.} \) \( \theta = 30^\circ \)
\( A_1 = 387 \text{ sq. ft.} \) \( A_2 = 540 \text{ sq. ft.} \) \( n = 0.014 \)
\( D_{2n} = 10.25 \text{ ft.} \)

DETERMINE POINT OF CONTROL:

\[
M_{2c} = 7540
\]

\[
M_{1n} + M_3 \cos \theta + \frac{1}{2} (A_{1n} + A_{2c})(Z + D_{1n} - D_{2c}) = 9770 + 108 - 1840
\]
\[
= 8038
\]

\( M_{2c} \) is the lower value;

\( D_1 = D_{1n} = 10.0 \text{ ft.}; \) DETERMINE \( D_2 \).

FORMULA:

\[
\frac{1}{2} (A_1 + A_2)(Z + D_1 - D_2) = \frac{Q_2^2}{A_2 g} - \frac{Q_1^2}{A_1 g} - \frac{Q_3^2 \cos \theta}{A_3 g}
\]

\[
(193 + \frac{1}{2} A_2)(10.0 - D_2) = \frac{4,070,000}{A_2} - 9880
\]

SOLUTION:

<table>
<thead>
<tr>
<th>( D_2 )</th>
<th>( A_2 )</th>
<th>( \Sigma P )</th>
<th>( \Sigma M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.5</td>
<td>406</td>
<td>-198</td>
<td>120</td>
</tr>
<tr>
<td>11.0</td>
<td>425</td>
<td>-406</td>
<td>-290</td>
</tr>
<tr>
<td>11.5</td>
<td>445</td>
<td>-624</td>
<td>-730</td>
</tr>
</tbody>
</table>

PLOT \( D_2 \) VERSUS \( \Sigma P \) and \( \Sigma M \):

\( D_2 = 11.25 \text{ ft.} \)

\( \Delta y = Z + D_1 - D_2 = 0 + 10.0 - 11.25 = 1.25 \text{ ft.} \)
F. Examples (Continued)

DETERMINE WATER SURFACE PROFILE FROM $D_2(11.25$ ft.) to $D_{2n}(10.25$ ft.).

3. Circular Section - Subcritical Flow

Case F: Flow is subcritical upstream and downstream, $Z = 0.5$ ft., $D_2 = D_{2n}$; DETERMINE $D_1$.

**Given:**
- $Q_1 = 200$ cfs
- $Q_2 = 250$ cfs
- $Q_3 = 50$ cfs
- $d_1 = 5.5$ ft.
- $d_2 = 6.0$ ft.
- $d_3 = 2.5$ ft.
- $S_1 = 0.0044$
- $S_2 = 0.0036$
- $A_3 = 4.91$ sq.ft.
- $D_{1n} = 3.99$ ft.
- $D_{2n} = 4.83$ ft.
- $A_{2n} = 24.4$ sq.ft.
- $\theta = 30^\circ$
- $n = 0.013$

**Formulas:**

$$\Sigma P = \Sigma M$$

$$\frac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2^2}{A_2g} - \frac{Q_1^2}{A_1g} - \frac{Q_3^2\cos\theta}{A_3g}$$

$$\frac{1}{2}(A_1+24.4)(0.5+D_1-4.83) = \frac{(250)^2}{24.4g} - \frac{(200)^2}{A_1g} - \frac{(50)^2(0.866)}{4.91g}$$

$$(\frac{1}{2}A_1+12.2)(D_1-4.33) = 79.6 - \frac{1242}{A_1} - 13.7$$
F. Examples (Continued)

SOLUTION:

<table>
<thead>
<tr>
<th>D₁</th>
<th>A₁</th>
<th>ΣP</th>
<th>ΣM</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>20.85</td>
<td>3.85</td>
<td>6.2</td>
</tr>
<tr>
<td>4.7</td>
<td>21.7</td>
<td>8.53</td>
<td>8.7</td>
</tr>
<tr>
<td>4.9</td>
<td>22.2</td>
<td>13.28</td>
<td>9.8</td>
</tr>
</tbody>
</table>

PLOT D₁ VERSUS ΣP and ΣM:

D₁ = 4.71 ft.

Δy = Z + D₁ - D₂ = 0.5 + 4.71 - 4.83 = 0.38 ft.

Case G: Determine required invert slopes and Z value to maintain 3/4 depth of flow:

\[
\frac{D₁}{d₁} = \frac{D₂}{d₂} = 0.75
\]

GIVEN: Q₁ = 200 cfs  Q₂ = 250 cfs  Q₃ = 50 cfs  
\(d₁ = 5.5 \text{ ft.}\)  \(d₂ = 6.0 \text{ ft.}\)  \(d₃ = 2.5\)  
\(D₁ = 4.13 \text{ ft.}\)  \(D₂ = 4.50 \text{ ft.}\)  \(A₃ = 4.91 \text{ sq.ft.}\)  
\(A₁ = 19.1 \text{ sq.ft.}\)  \(A₂ = 22.8 \text{ sq.ft.}\)  \(θ = 30^°\)  
\(n = 0.013\)

USING MANNINGS EQUATION, \(S₁ = 0.00428, S₂ = 0.00419\).
HYDRAULIC ANALYSIS OF JUNCTIONS

F. Examples (Continued)

FORMULA AND SOLUTION:

\[ \sum P = \sum M \]

\[
\frac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2^2}{A_2g} - \frac{Q_1^2}{A_1g} - \frac{Q_3\cos\theta}{A_3g}
\]

\[
\frac{1}{2}(19.1+22.8)(Z+4.13-4.50) = \frac{(250)^2}{22.8g} - \frac{(200)^2}{19.1g} - \frac{(50)^2}{4.91g}
\]

\[
(21.0)(Z-0.37) = 85.2-65.0-13.7
\]

\[ Z = 0.68 \text{ ft.} \]

\[ \Delta y = Z+D_1-D_2 = 0.68+4.13-4.50 = 0.31 \text{ ft.} \]

4. Circular Section - Supercritical Flow

Case H: Supercritical flow upstream and downstream
\[ Z = 0.5 \text{ ft.} \]

GIVEN: \( Q_1 = 200 \text{ cfs} \quad Q_2 = 250 \text{ cfs} \quad Q_3 = 50 \text{ cfs} \)
\[ d_1 = 5.5 \text{ ft.} \quad d_2 = 6.0 \text{ sq.ft.} \quad d_3 = 2.5 \text{ ft.} \]
\[ S_1 = 0.0055 \quad S_2 = 0.0050 \quad A_3 = 4.91 \text{ sq.ft.} \]
\[ D_{1n}= 3.71 \text{ ft.} \quad D_{2c}= 4.33 \text{ ft.} \quad \theta = 30^\circ \]
\[ A_{1n}= 17.1 \text{ sq.ft.} \quad A_{2c}= 21.8 \text{ sq.ft.} \quad n = 0.013 \]
\[ D_{2n}= 4.17 \text{ ft.} \]
F. Examples (Continued)

\[ M_{2c} = \frac{Q_2^2}{A_{2c}g} = \frac{(250)^2}{21.8g} = 89.0 \]

\[ M_{1n} + M_3 \cos \theta + \frac{1}{2}(A_{1n} + A_{2c})(Z + D_{1n} - D_{2c}) = \frac{(200)^2}{17.1g} + \frac{(50)^2(0.866)}{4.91g} + \frac{1}{2}(17.1 + 21.8)(0.5 + 3.71 - 4.33) = 84.1 \]

\( M_{2c} \) is larger \( (89 > 84.1) \)

A hydraulic jump will form upstream of the junction.

\[ D_2 = D_{2c}, \text{ determine } D_1. \]

**FORMULA:**

\[ \Sigma P = \Sigma M \]

\[ \frac{1}{2}(A_1 + A_2)(Z + D_1 - D_2) = \frac{Q_2^2}{A_2g} - \frac{Q_1^2}{A_1g} - \frac{Q_3^2 \cos \theta}{A_3g} \]

\[ (\frac{1}{2}A_1 + 10.9)(D_1 - 3.83) = 75.3 - \frac{1242}{A_1} \]

**SOLUTION:**

<table>
<thead>
<tr>
<th>( D_1 )</th>
<th>( A_1 )</th>
<th>( \Sigma P )</th>
<th>( \Sigma M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.25</td>
<td>19.7</td>
<td>8.7</td>
<td>12.2</td>
</tr>
<tr>
<td>4.50</td>
<td>20.8</td>
<td>14.3</td>
<td>15.7</td>
</tr>
<tr>
<td>4.75</td>
<td>21.8</td>
<td>20.0</td>
<td>18.3</td>
</tr>
</tbody>
</table>

PLOT \( D_1 \) versus \( \Sigma P \) and \( \Sigma M \):

\[ D_1 = 4.65 \text{ ft.} \]

\[ \Delta y = Z + D_1 - D_{2c} = 0.5 + 4.65 - 4.33 = 0.82 \text{ ft.} \]
F. Examples (Continued)

Case I: Solve for the value of Z so that a hydraulic jump cannot occur. \( D_1 = D_{1n} \), Maximum \( D_2 = D_{2c} \).

\[
\frac{M_{2c}}{M_{2c}} = M_{1n} + M_3 \cos \theta + \frac{1}{2} (A_{1n} + A_{2c})(Z + D_{1n} - D_{2c})
\]

\[
Q_{2c}^2 = \frac{Q_1^2}{A_{1n} \sin \theta} + \frac{Q_3^2 \cos \theta}{A_{3n}} + \frac{1}{2} (A_{1n} + A_{2c})(Z + D_{1n} - D_{2c})
\]

\[
89.0 = 72.7 + 13.7 + \frac{1}{2}(17.1 + 21.8)(Z + 3.71 - 4.33)
\]

\[
Z = 0.75 \text{ ft.}
\]

**ALTERNATE SOLUTION:** CONSIDER \( D_2 = D_{2n} = 4.17 \) ft.,

\[
A_{2n} = 21.0 \text{ sq. ft.}
\]

\[
M_{2n} = M_{1n} + M_3 \cos \theta + \frac{1}{2} (A_{1n} + A_{2n})(Z + D_{1n} - D_{2n})
\]

\[
92.4 = 72.7 + 13.7 + 19.0Z - 8.8
\]

\[
Z = 0.78 \text{ ft.}
\]

\[
\Delta y = Z + D_{1n} - D_{2n} = 0.78 + 3.71 - 4.17 = 0.32 \text{ ft.}
\]
F. Examples (Continued)

Case J: Supercritical flow upstream and downstream; Z = 0.75 ft.

\[ \begin{align*}
GIVEN: & \quad Q_1 = 200 \text{ cfs} \quad Q_2 = 250 \text{ cfs} \quad Q_3 = 50 \text{ cfs} \\
& \quad d_1 = 5.5 \text{ ft.} \quad d_2 = 6.0 \text{ ft.} \quad d_3 = 2.5 \text{ ft.} \\
& \quad S_1 = 0.0065 \quad S_2 = 0.0050 \quad A_3 = 4.91 \text{ sq.ft.} \\
& \quad D_{1n} = 3.50 \text{ ft.} \quad D_{2c} = 4.33 \text{ ft.} \quad \theta = 30^\circ \\
& \quad A_{1n} = 15.95 \text{ sq.ft.} \quad A_{2c} = 21.8 \text{ sq.ft.} \quad n = 0.013 \\
& \quad D_{2n} = 4.17 \text{ ft.}
\end{align*} \]

DETERMINE POINT OF CONTROL:

\[ \begin{align*}
M_{2c} &= 89.0 \\
M_{1n} + M_3 \cos \theta &+ \frac{1}{2}(A_{1n} + A_{2c})(Z + D_{1n} - D_{2c}) \\
&= \frac{(200)^2}{15.95g} + \frac{(50)^2(0.866)}{4.91g} \\
&\quad + \frac{1}{2}(15.95 + 21.8)(0.75 + 3.50 - 4.33) \\
&= 90.2 \\
M_{2c} \text{ is smaller } (89 < 90.2)
\end{align*} \]

\[ D_1 = D_{1n}, \text{ DETERMINE } D_2. \]
HYDRAULIC ANALYSIS OF JUNCTIONS

F. Examples (Continued)

FORMULA:

\[ \Sigma P = \Sigma M \]

\[ \frac{1}{2}(A_1+A_2)(Z+D_1-D_2) = \frac{Q_2^2}{A_2g} - \frac{Q_1^2}{A_1g} - \frac{Q_3^2 \cos \theta}{A_3g} \]

\[ (8.0+\frac{1}{2}A_2)(4.25-D_2) = \frac{1940}{A_2} - 91.7 \]

SOLUTION:

<table>
<thead>
<tr>
<th>(D_2)</th>
<th>(A_2)</th>
<th>(\Sigma P)</th>
<th>(\Sigma M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00</td>
<td>20.0</td>
<td>4.50</td>
<td>5.3</td>
</tr>
<tr>
<td>4.25</td>
<td>21.4</td>
<td>0.00</td>
<td>-1.0</td>
</tr>
<tr>
<td>4.33</td>
<td>21.8</td>
<td>-1.30</td>
<td>-2.7</td>
</tr>
</tbody>
</table>

PLOT \(D_2\) VERSUS \(\Sigma P\) and \(\Sigma M\):

\[ D_2 = 4.12 \text{ ft.} \]

\[ Ay = Z+D_1-D_2 = 0.75+3.50-4.12 = 0.13 \text{ ft.} \]

IV PRESSURE FLOW

A. Criteria for Junction

A junction structure as generally constructed includes a junction plus transition structures on either side. The loss across the structure will include transition losses as well as the junction loss. The hydraulic grade for the lateral is the same as for the main line where the lateral joins the upstream end of the main line (Point 0). The transition losses are very minor and the junction structure losses may be evaluated by using the physical properties at the ends of the junction structure.
A. Criteria for Junction (Continued)

Tr₁ and Tr₂ are transition sections either side of junction.

B. General Conditions

Junction Loss

\[ \Delta y(\text{AVERAGE AREA}) = Q_2 V_2' - Q_1 V_1' - Q_3 V_3 \cos \theta \]

\[ + \frac{1}{2} (A_1' + A_2') L_j \left[ \frac{1}{2} (S_{f1}' + S_{f2}') L_j \right] \]

\[ \Delta y_j = \frac{Q_2 V_2' - Q_1 V_1' - Q_3 V_3' \cos \theta}{\frac{1}{2} (A_1' + A_2')} + \frac{1}{2} L_j (S_{f1}' + S_{f2}') \]

\[ h_j = \Delta y + h_{v1}' - h_{v2}' \]

Transition Loss (Enlargers)

Based on tests by Gibson (Standards of the Hydraulic Institute).

\[ h_{tr} = \frac{K(V_1 - V_2)^2}{2g} \]

\[ K = 3.50 \ (\tan \phi/2)^{1.22} \]
B. General Conditions (Continued)

For $\phi = 11^\circ 30'$

$\tan \frac{\phi}{2} = 0.100$

$K = 0.211$

$K = \frac{0.211}{64.4} = 0.0032$ for $\phi = 11^\circ 30'$

$h_{tr1} = 0.0032 (V_1-V_1')^2 + \frac{1}{2} (Sf_1+Sf_1')L_1$

$h_{tr2} = 0.0032 (V_2'-V_2)^2 + \frac{1}{2} (Sf_2+Sf_2')L_2$

Junction Structure Loss

$h_{STRUCT} = h_j + h_{tr1} + h_{tr2}$

$h_{STRUCT} = \Delta y + h_{V1}' - h_{V2} + h_{tr1} + h_{tr2}$

Since $h_{tr}$ values are small, the transitions can be ignored.

Use all values at $d_1$ and $d_2$.

$Q_2V_2 - Q_1V_1 - Q_3V_3 \cos \theta$

$\Delta y = \frac{Q_2V_2 - Q_1V_1 - Q_3V_3 \cos \theta}{\frac{1}{2}(A_1+A_2)g} + \frac{1}{2} (Sf_1+Sf_2)L$

$h_{STRUCT} = \Delta y + h_{V1}' - h_{V2}$

To determine lateral hydraulic grade

$H.G. Lateral = H.G.(l) + h_{V1}' - (Sf_1')L_1$

C. Derivation of Formula

1. Rectangular Section

Derivation of Equation (4), by D. Thompson.

Rectangular Box with expansion, friction ignored.
C. Derivation of Formula (Continued)

\[ P_1 - P_2 + P_1 - P_s + P_w = M_2 - M_1 - M_3 \cos \theta \]

\[ X + d_2 = d_1 + Z \quad X = Z + d_1 - d_2 \]

\[ \Delta y + D_2 = D_1 + Z \quad \Delta y = Z + D_1 - D_2 \]

\[ P_1 = b_1 d_1 (D_1 - \frac{1}{2} d_1) \]

\[ P_2 = b_2 d_2 (D_2 - \frac{1}{2} d_2) \]

\[ P_1 = \frac{1}{6} Z \left[ b_1 D_1 + 4 \left( \frac{1}{2} \right) (b_1 + b_2) \left( \frac{1}{2} \right) (D_1 + D_2) + b_2 D_2 \right] \]

\[ P_s = \frac{1}{6} X \left[ b_1 (D_1 - d_1) + 4 \left( \frac{1}{2} \right) (b_1 + b_2) \left( \frac{1}{2} \right) (D_1 - d_1 + D_2 - d_2) \right. \]

\[ + b_2 (D_2 - d_2) \]

\[ P_w = \frac{1}{6} \left( \frac{5}{2} \right) (b_2 - b_1) \left[ \frac{1}{3} (D_1 + D_1 - d_1) d_1 + 4 \left( \frac{1}{2} \right) (d_1 + d_2) \left( \frac{1}{3} \right) \right. \]

\[ \left. \left( \frac{1}{3} (2D_1 - d_1) + \frac{1}{3} (2D_2 - d_2) + \frac{1}{3} (D_2 + D_2 - d_2) d_2 \right) \right]^2 \]

\[ \Sigma P = \Delta y (\text{Average Area}) \]

\[ \Sigma P = P_1 - P_2 + P_1 - P_s + P_w \]

\[ P_1 = b_1 d_1 D_1 - \frac{1}{2} (b_1 d_1)^2 = \frac{1}{6} (6b_1 d_1 D_1 - 3b_1 d_1^2) \]
C. Derivation of Formula (Continued)

\[ P_2 = b_2 d_2 D_2 - \frac{3}{2} (b_2 d_2^2) = \frac{1}{6} \left( 6 b_2 d_2 D_2 - 3 b_2 d_2^2 \right) \]

\[ P_1 = \frac{1}{6} Z \left( b_1 D_1 + b_1 D_2 + b_2 D_1 + b_2 D_2 \right) + b_2 D_1 - b_2 d_2 + b_2 D_2 - b_2 d_2 + b_2 D_2 - b_2 d_2 \]

\[ = \frac{1}{6} Z \left( 2 b_1 D_1 + b_1 D_2 + b_2 D_1 + 2 b_2 D_2 \right) \]

\[ p_o = \frac{1}{6} (Z + d_1 - d_2) \left( b_1 D_1 - b_1 d_1 + b_1 D_2 - b_1 d_1 + b_2 D_1 - b_2 d_1 + b_2 D_2 - b_2 d_2 \right) \]

\[ = \frac{1}{6} (Z + d_1 + d_2) \left( 2 b_1 D_1 - 2 b_1 d_1 + b_1 D_2 - b_1 d_2 + b_2 D_1 - b_2 d_1 + 2 b_2 D_2 - 2 b_2 d_2 \right) \]

\[ = \frac{1}{6} (2 b_1 D_1 Z - 2 b_1 d_1 Z + b_1 D_2 Z - b_1 d_2 Z + b_2 D_1 Z - b_2 d_1 Z + 2 b_2 D_2 Z - 2 b_2 d_2 Z) \]

\[ p_w = \frac{1}{6} (b_2 - b_1) \left( D_1 d_1 - d_1 d_1^2 + d_1 D_1 - d_1 d_1^2 + d_1 D_2 - d_1 d_2^2 \right) \]

\[ = \frac{1}{6} (b_2 - b_1) \left( 2 d_1 D_1 - d_1 D_1^2 + d_1 D_2 - d_1 d_2 + d_2 D_1 + 2 d_2 D_2 - d_2^2 \right) \]
C. Derivation of Formula (Continued)

\[ \xi P = \frac{1}{6} (6b_1 d_1 D_1 - 3b_1 d_1^2 - 6b_2 D_2 + 3b_2 d_2^2 + 2b_1 D_1 Z + b_1 D_2 Z + b_2 D_1 Z + 2b_2 D_2 Z - 2b_1 D_1 Z - b_1 D_2 Z - b_2 D_1 Z - 2b_2 D_2 Z + 2b_1 d_1 Z + b_1 d_2 Z + b_2 d_1 Z + 2b_2 d_2 Z - 2b_1 d_1 D_1 + 2b_1 d_1^2 - b_1 d_1 D_2 - b_1 d_1 d_2 - 2b_2 d_1 D_2 + b_2 d_1 d_2 + 2b_1 d_2 D_1 + b_1 d_2 D_2 - b_1 d_2^2 + b_2 D_1 d_2 + 2b_2 D_2 d_2 - 2b_2 d_2^2 + 2b_2 d_1 D_1 - b_2 d_1^2 + b_2 d_1 D_2 - 2b_2 d_2 D_1 + 2b_2 d_2 D_2 - 2b_2 d_2^2 - 2b_1 d_1 D_1 + b_1 d_1^2 - b_1 d_1 D_2 + b_1 d_1 d_2 - b_1 d_2 D_1 - 2b_1 d_2 D_2 + b_1 d_2^2) \]

\[ \Delta y (AVERAGE AREA) = \left( Z + D_1 - D_2 \right) \frac{1}{6} \left[ b_1 d_1 + 4 \left( \frac{1}{2} (b_1 + b_2) \right) \left( \frac{1}{2} (d_1 + d_2) + b_2 d_2 \right) \right] \]

\[ = \frac{1}{6} (b_1 d_1 + b_1 d_1 + b_1 d_2 + b_2 d_1 + b_2 d_2 + b_2 d_2) (Z + D_1 - D_2) \]

\[ = \frac{1}{6} (2b_1 d_1 + b_1 d_2 + b_2 d_1 + 2b_2 d_2) (Z + D_1 - D_2) \]

- 32 -
C. Derivation of Formula (Continued)

\[ \Delta y(\text{AVERAGE AREA}) = \frac{1}{6}(2b_1d_1D_1 - 2b_1d_1D_2 - 2b_2d_2D_2 + 2b_2d_2D_1 + b_1d_2D_1 + b_2d_1D_1 - b_1d_2D_2 - b_2d_1D_2 + 2b_1d_1Z + b_1d_2Z + b_2d_1Z + 2b_2d_2Z) \]

\[ \Sigma P = \Delta y(\text{AVERAGE AREA}) = \Sigma M = M_2 - M_1 - M_3 \cos \theta \]

2. Circular Section

Derivation of Equation (4), by D. Thompson.

Circular conduit with expansion, friction ignored.

\[ P_1 = A_1 \bar{V}_1 = \pi/4d_1^2(D_1 - \frac{1}{3}d_1) = \pi/8(2D_1d_1^2 - d_1^3) \]

\[ P_2 = A_2 \bar{V}_2 = \pi/4d_2^2(D_2 - \frac{1}{3}d_2) = \pi/8(2D_2d_2^2 - d_2^3) \]

\[ P_1 = 0 \]

\[ P_W = A_W \bar{V}_W \]

\[ A_W = \text{VERTICAL PROJECTION OF WALL} = A_2 - A_1 \]
C. Derivation of Formula (Continued)

\[ \bar{y}_w = \text{C.G. of vertical projection of transition plus average distance of H.G. above transition soffit.} \]

\[ A_w = \pi/4(d_2^2-d_1^2) \]

\[ \text{C.G. from soffit} = \frac{\pi/4(d_2^2)(d_2^2)-\pi/4(d_1^2)(d_1^2)}{\pi/4(d_2^2-d_1^2)} \]

\[ = \frac{d_2^3-d_1^3}{2(d_2^3-d_1^3)} \]

\[ \bar{y}_w = \frac{d_2^3-d_1^3}{2(d_2^2-d_1^2)} + \frac{1}{2}(D_1-d_1)+\frac{1}{2}(D_2-d_2) \]

\[ P_w = A_w \bar{y}_w \]

\[ = \pi/4(d_2^2-d_1^2)\left(\frac{d_2^3-d_1^3}{2(d_2^2-d_1^2)} + \frac{D_1+D_2-d_1-d_2}{2}\right) \]

\[ \Sigma P = P_1+P_w-P_2 \]

\[ = \pi/8(2D_1d_1^2-d_1^3+d_2^3-d_1^3+D_1d_2^2+D_2d_2^2-D_1d_1^2-D_2d_2^2-d_1d_2^2-d_1^3-d_2^3+d_2d_1^2-2D_2d_2^2+d_2^3) \]

\[ \Delta y = (D_1-d_1)-(D_2-d_2); \]

\[ \text{Average area} = \frac{1}{2}(A_1+A_2) = \pi/8(d_1^2+d_2^2) \]

\[ \Delta y(\text{Average area}) = \pi/8(D_1d_1^2-D_2d_1^2-d_1^3+d_2d_1^2+D_1d_2^2 \]

\[ -D_2d_1^2-d_1^2+d_2^3) \]

\[ \text{By inspection:} \Delta y(\text{Average area}) = \Sigma P = \Sigma \text{Moments} \]
C. Derivation of Formula (Continued)

\[ \Delta y \text{(AVG AREA)} = \frac{Q_2 V_2 - Q_1 V_1 - Q_3 V_3 \cos \theta}{g} \]

\[ = \frac{Q_2^2}{A_2 g} - \frac{Q_1^2}{A_1 g} - \frac{Q_3^2 \cos \theta}{A_3 g} \]

D. Example: Circular Section

Determine hydraulic and energy gradient across junction and across structure.

Pipe 1
- \( d_1 = 4.5' (54") \)
- \( Q_1 = 223 \text{ cfs} \)
- \( A_1 = 15.9 \text{ sq.ft.} \)
- \( V_1 = 14.0 \text{ fps} \)
- \( h_{v1} = 3.04 \text{ ft.} \)
- \( S_{f1} = 0.0128 \)

Pipe 2
- \( d_2 = 5.5' (66") \)
- \( Q_2 = 380 \text{ cfs} \)
- \( A_2 = 23.8 \text{ sq.ft.} \)
- \( V_2 = 16.0 \text{ fps} \)
- \( h_{v2} = 3.98 \text{ ft.} \)
- \( S_{f2} = 0.0128 \)

Pipe 3
- \( d_3 = 3.5' (42") \)
- \( Q_3 = 157 \text{ cfs} \)
- \( A_3 = 9.6 \text{ sq.ft.} \)
- \( V_3 = 16.4 \text{ fps} \)
- \( h_{v3} = 4.17 \text{ ft.} \)
- \( S_{f3} = 0.0246 \)

\( \theta = 45^\circ \)
- \( n = 0.013 \)
- \( L_1 = 8.8 \text{ ft.} \)
- \( L_2 = 2.6 \text{ ft.} \)
- \( L_3 = 4.7 \text{ ft.} \)

\( \delta \)
D. Example: Circular Section (Continued)

1. Transition Losses Considered

\[ \Delta y_j (\text{AVERAGE AREA}) = \frac{Q_2 V_2' - Q_1 V_1' - Q_3 V_3 \cos \theta}{A} \]

\[ + \frac{1}{2}(S_{f1} + S_{f2'}) L_j \left( \frac{1}{2} \right)(A_1' + A_2') \]

\[ \Delta y_j = \frac{Q_2 V_2' - Q_1 V_1' - Q_3 V_3 \cos \theta}{\frac{1}{2}(A_1' + A_2') g} + \frac{1}{2}(S_{f1} + S_{f2'}) L_j \]

\[ \Delta y_j = \frac{380(17.3) - 223(12.3) - 157(16.4)(.707)}{\frac{1}{2}(18.1 + 22.1) 32.2} \]

\[ + \frac{1}{2}(0.0092 + 0.0156) \]

\[ \Delta y_j = 3.1584' \]

\[ h_j = \Delta y_j + h_{v_1}' - h_{v_2}' = 3.158 + 2.35 - 4.62 = 0.888' \]

\[ h_{tr1} = 0.0032(V_1 - V_1')^2 + \frac{1}{2}(S_{f1} + S_{f1'}) L_1 \]

\[ = 0.0032(14.0 - 12.3)^2 + \frac{1}{2}(0.0128 + 0.0092)(2.6) \]

\[ = 0.0378 \]

\[ h_{tr2} = 0.0032(V_2' - V_2)^2 + \frac{1}{2}(S_{f2} + S_{f2'}) L_2 \]

\[ = 0.0032(17.3 - 16.0)^2 + \frac{1}{2}(0.0128 + 0.0156)(1.5) \]

\[ = 0.0267 \]

Computing Hydraulic and Energy Grade

<table>
<thead>
<tr>
<th>Assume H.G. at Pipe 2 = 100.00</th>
<th>H.G.</th>
<th>E.G.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.G. = 100.00 + 3.980</td>
<td>103.980</td>
<td>103.980</td>
</tr>
</tbody>
</table>

| AT d_2': E.G. = 103.980 + 0.0267 | 104.007 |
| H.G. = 104.007 - 4.620 | 99.387 |

| AT d_1': H.G. = 99.387 + 3.158 | 102.545 |
| E.G. = 102.545 + 2.350 | 104.895 |

| AT d_1: E.G. = 104.895 + 0.0378 | 104.933 |
| H.G. = 104.933 - 3.640 | 101.893 |
D. Example: Circular Section (Continued)

2. Transition Losses Ignored

Using $d_1$ and $d_2$

$$\Delta y = \frac{Q_2V_2 - Q_1V_1 - Q_3V_3\cos\theta}{\frac{1}{2}(A_1 + A_2)E} + \frac{1}{2}(Sf_1 + Sf_2)L$$

$$\Delta y = \frac{380(16.0) - 223(14.0) - 157(16.4)0.707}{\frac{1}{2}(15.9 + 23.8)32.2} + \frac{1}{2}(0.0128 + 0.0128)(8.8)$$

$$\Delta y = \frac{1138}{32.2(19.85)} + 8.8(0.0128) = 1.8930$$

AT $d_2$:

H.G. = 100.00
E.G. = 100.00 + 3.980 = 103.980

AT $d_1$:

H.G. = 100.00 + 1.893 = 101.893
E.G. = 101.893 + 3.040 = 104.933

Difference: 104.933 - 104.933 = 0.000

AT $d_1'$:

Lateral H.G. = H.G.(1) + $h_v_1$ - $h_v_1'$ - $Sf_1'L_1$

Lateral H.G. = 101.893 + 3.040 - 2.350 - 0.024 = 102.559

Difference: 102.559 - 102.545 = 0.014

Values from Methods 1 and 2 are approximately equal. Method 2 using the average end areas should be used in determining the junction loss. The transition losses are small enough to be ignored.